Engineering Mechanics.

Engineering Mechanics

Definition of Mechanics

It is defined as that branch of science, which describes and predicts the conditions of rest and motion of bodies under the action of forces. ~~Engineerin~~ Engineering mechanics applies the principle of mechanics to design, taking into account the effects of forces.

- Mechanics
  - Mechanics of fluids
  - Mechanics of deformable body or Strength of materials
  - Mechanics of Rigid bodies
    - Statics
    - Dynamics

Statics

It is a branch of mechanics that deals with bodies at rest or forces in equilibrium.

Dynamics

It is a branch of mechanics which is concerned with the motion of bodies under the action of the forces.

Rigid body

A rigid body is defined as a material body which, when subjected to a system of external stimuli, does not change its geometry and size.
usually in a rigid body the generalized deformation ~~is~~ (elongation, contraction and twist) is ignored during analysis because either the deformation is too small or ~~is its~~ has very little effect on its motion and equilibrium.

Force

Force is the action of one body on another. It may be defined as an action which ~~are~~ tends to change the state of rest or of uniform motion of body

For representing force acting on the body, the magnitude of force, its point of action and direction of its action should be known. There are different types of forces such as gravitational, frictional, magnetic, inertia or those caused by mass and acceleration.

$$F = ma = \text{mass} \times \frac{\text{Distance}}{(\text{Time})^2}$$

SI unit of force is Newton (N)

One Newton force is defined as that which gives an ~~accl~~ acceleration of $1 m/s^2$ to a body of mass of 1 kg in the direction of force

Thus, $1N = 1kg \times 1 m^2/s^2 = 1 kg \cdot m/s^2$

The three requisistes for representing the force acting on the body are:

1) Magnitude of force
2) It points of action
3) Direction of its action

Characteristics of a Force

To know the effect of force on a body, the following elements of force should be known.

1) Magnitude (ie. 2N, 5kN, 10kN etc)
2) Direction or line of action
3) Sense or nature (Push or pull)
4) Point of application

Characteristics

1) Force are due to an interaction of at least two objects.

2) It may change the state of motion of an object.

3) It may change the shape of an objects.

4) Forces applied on an object in the same direction add to one another and resultant is in the same direction.

5) When the forces are applied on an object in the opposite direction then their resultant or net force is the difference between these opposing forces and its resulting direction is the as that of larger force.

6) If the two forces acting on object are equal in magnitude but opposite in direction, then the net force acting on the body is zero.

7) It is a vector quantity hence they should be specified by giving its magnitude and the direction

8) If the magnitude or the direction or both changes, then the effect of force also changes.

Effects of a force

A force may produce the following effects in a body, on which it acts.

i) It may change the motion, that is if a body is at rest, the force may set it in motion. And if the body is already in motion, the force may accelarate or retard it.

ii) It may retard the forces, already acting on a body, thus bringing it to rest or in equilibrium.

iii) It may give rise to the internal stresses in the body, on which it acts.

Force field

Force field indicates the presence of a force as a function of Cartesian coordinates (1D, 2D, or 3D

• (i) Steady linear force field:

This force field is restricted along a line and it is a function of one dimension (1D) only.

Hence $F = F(x)$

(ii) Steady planar force field

The variation of force field occurs over a plane and force field is a function of two dimensions.

Thus $F = F(x, y)$

iii) Steady spatial force field

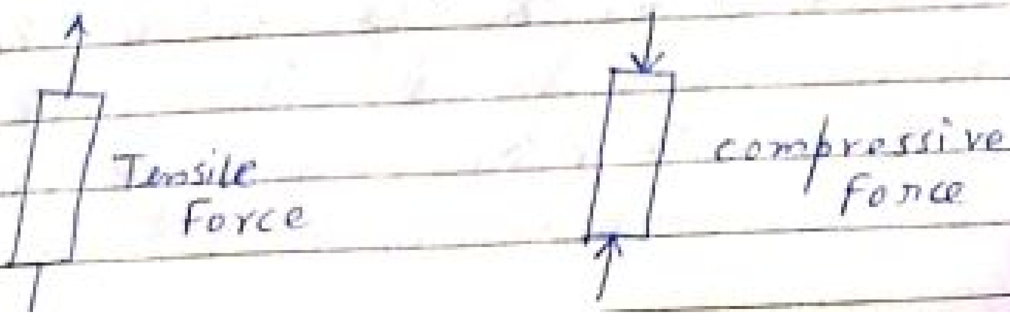
This variation of force spreads over a space and force field is a function of three dimensions.

Thus, $F = F(x, y, z)$

Classification of force

- Tensile and compressive force: Depending on the nature of action, forces are either tensile or compressive. At any point of the body, tensile force acts away from the point to impart an elongative effect.

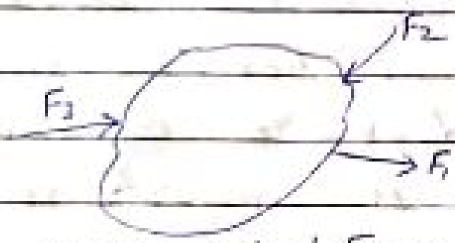
  On the other hand, compressive force acts towards the point of the body to create a compressive effect.



• Concentrated and distributed force:

Depending on the ~~place~~ place of action on a body, forces are classified as concentrated force or distributed force.

If the forces act on a particular point on the body, it is defined as a concentrated force. If the force acts distributed over a ~~distributed~~ definite distance then the force is called distributed force.



concentrated Force

Distributed Force

Force System

A force system is a collection of forces acting at specified locations and may also include couples. Force system is simply a term used to denote a group of forces. It can be classified into the following types.

• Collinear and non-Collinear force systems

If all the forces have lines of action lying along a particular line, a collinear force system develops. In a non-collinear force system, all the forces have separate lines of action.

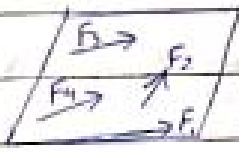
collinear force system

Non collinear force sys

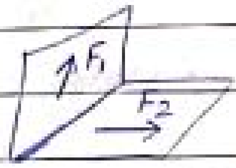
Collinear and non-collinear force system

Coplanar and non-coplanar force system

In a coplanar force system, all forces acting on a material body lie in the same plane (vertical, horizontal, inclined). On the other hand in a non-coplanar force system, there is no restriction and the forces may be either parallel or non parallel.



Coplanar



Non-coplanar

Concurrent and non-concurrent Force system

If all the forces acting on a material body either converge at or diverge from a particular point, then the force system is said to be concurrent.

In a non-concurrent force system, the forces neither converge at nor diverge from a particular point on the body.

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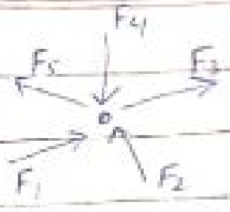
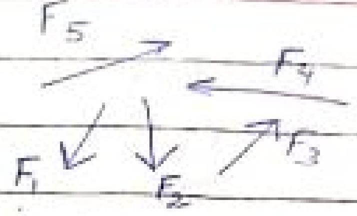


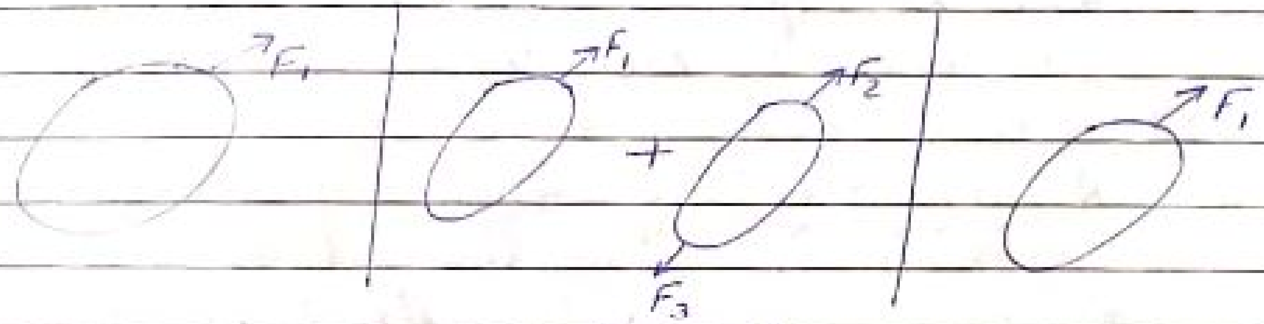
(Concurrent)



(Non concurrent)

Principle of superposition

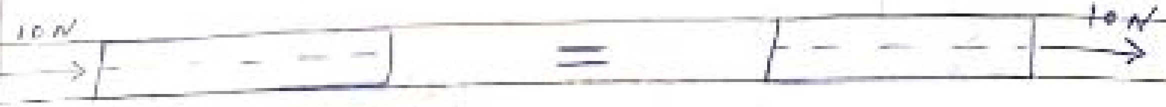
This principle state that the effect or action of a given system of forces on a non-deformable body will never get altered if another system of forces in equilibrium is added to or subtracted from it.



$\boxed{F_2 = F_3}$

Principle of Transmissibility of force

This principle state that the equilibrium or the motion of a rigid body remains unaltered if the point of application of any force vector acting on the body is displaced along the line of action of the force.



Free Body Diagram

A body may consist of more than one element and supports. Each element or support can be isolated from the rest of the system by incorporating the net effect of the remaining system through a set of forces. This diagram of the isolated element of a portions of the body along with the net effects of the system on it is called free body diagram.

Resultant Force

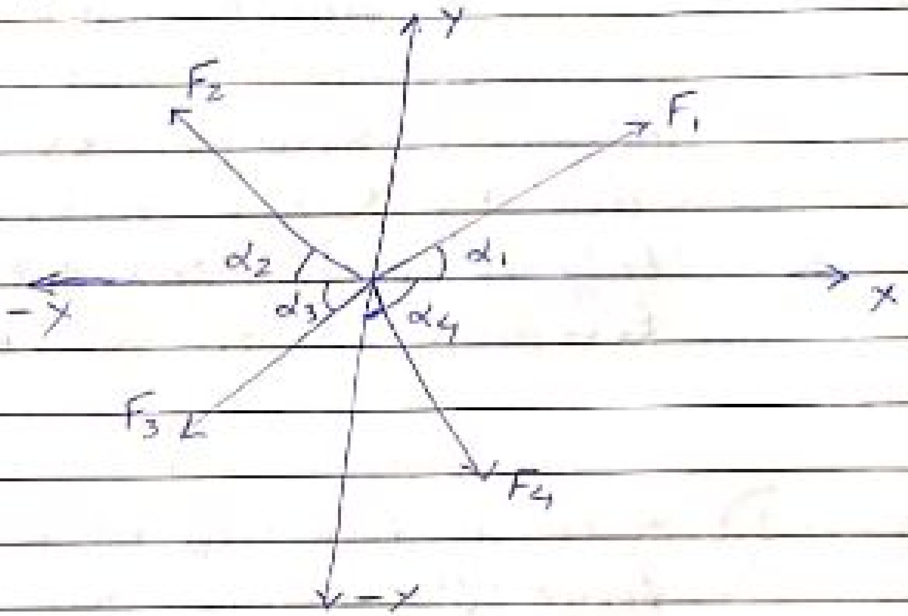
A single force which produces same effect on the body as the system of forces is called as resultant force.

Composition of Forces

Conversion of system of forces into an equivalent single force system is known as the composition forces.

The effect of single equivalent force will be same as the effect produce by number of forces action on a body.

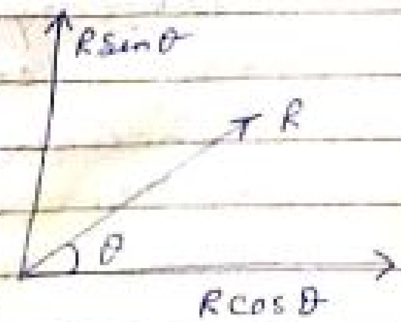
Let the forces F_1, F_2, F_3, F_4 are acting on a body in a plane making angle α_1, α_2, α_3 and α_4 with x-axis. Let R be the resultant force of all the forces acting at the point making an angle θ with the horizontal. Resolving the forces along x-axis and y-axis we get



$$\Sigma F_x = F_1 \cos\alpha_1 - F_2 \cos\alpha_2 - F_3 \cos\alpha_3 + F_4 \cos\alpha_4$$

$$\Sigma F_y = F_1 \sin\alpha_1 + F_2 \sin\alpha_2 - F_3 \sin\alpha_3 - F_4 \sin\alpha_4$$

Component of R along x-axis = $R\cos\theta$
Component of R along y-axis = $R\sin\theta$



$$R\cos\theta = \Sigma F_x$$
$$R\sin\theta = \Sigma F_y$$

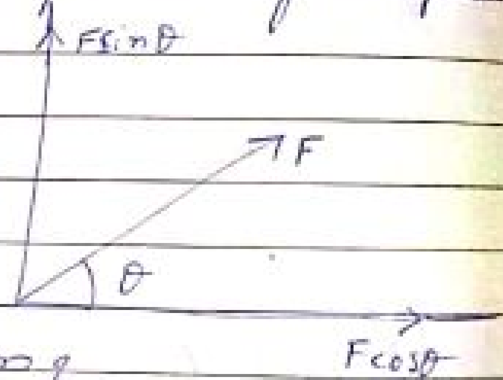
$$R^2(\sin^2\theta + \cos^2\theta) = (\Sigma F_x)^2 + (\Sigma F_y)^2$$

$$\boxed{R = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}}$$

$$\boxed{\tan\theta = \frac{\Sigma F_y}{\Sigma F_x}}$$

Resolution of Forces

Replacing force F by two forces along x and y axis acting on the same body is called resoluting / resolution of force. Resolution is the reverse process of composition.

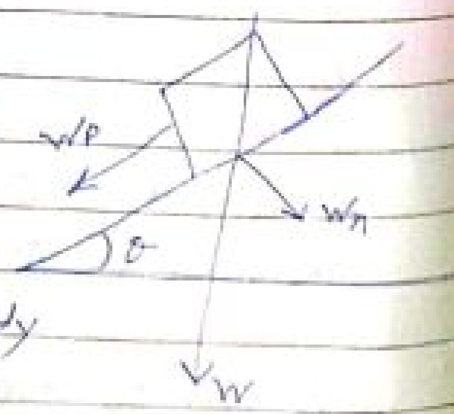


Case 1 A Force F acting at a point 'O' making angle θ with horizontal, Then its components along x and y axis are given by

$$F_x = F\cos\theta \text{ and } F_y = F\sin\theta$$

Case 2

The components of the ~~body~~ Force W when the body is on an inclined plane.



The components of the body force W are given by

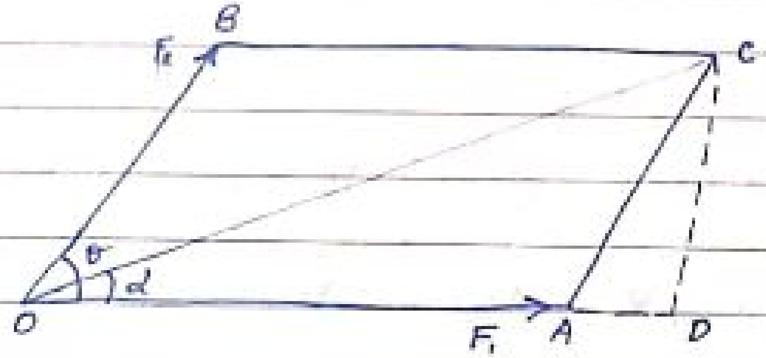
$$W_n = W\cos\theta \text{ and } W_p = W\sin\theta$$

where, W_n is normal component to the incline plane and W_p is parallel component to inclined plane.

Parallelogram Law of Forces

This law is used for finding the resultant of two forces acting at a point.

If two forces F_1 and F_2 are acting at a point and are represented in magnitude and direction by two sides of a parallelogram, then their resultant is represented by the diagonal of the parallelogram both in magnitude and direction



Let θ be the angle between the forces F_1 and F_2 and α be the angle made by R with force F_1.

From the above fig we can write

$$BC = OA = F_1$$

$$AC = OB = F_2$$

$$\angle BOA = \theta = \angle CAD$$

and $\triangle ODC$ and $\triangle ADC$ are right angle triangles.

From triangle ~~ODC~~ ADC,

$$AD = AC\cos\theta = F_2\cos\theta$$
$$CD = AC\sin\theta = F_2\sin\theta$$

From triangle ODC

$$OC^2 = OD^2 + CD^2 = (OA + AD)^2 + CD^2$$
$$R^2 = (F_1 + F_2\cos\theta)^2 + (F_2\sin\theta)^2$$
$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2\cos^2\theta + F_2^2\sin^2\theta$$
$$= F_1^2 + 2F_1F_2\cos\theta + F_2^2(\cos^2\theta + \sin^2\theta)$$
$$\therefore \quad R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2}$$

From ΔODC

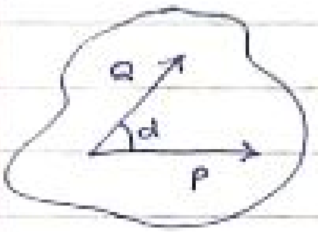
$$\tan\alpha = \frac{CD}{OD} = \frac{CD}{OA + AD} = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}$$

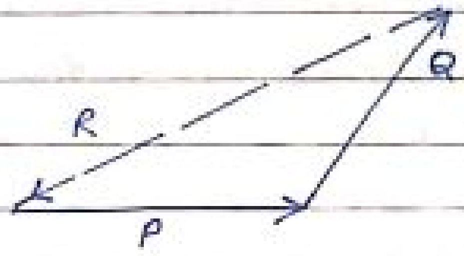
$$\boxed{R = \sqrt{F_1^2 + 2F_1F_2\cos\theta + F_2^2}}$$
$$\boxed{\tan\alpha = \frac{F_2\sin\theta}{F_1 + F_2\cos\theta}}$$

Traingle law of Forces

This law state that:

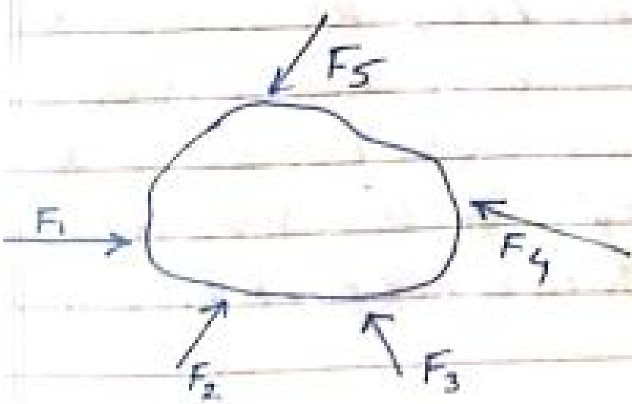
If two forces acting simultaneously on a body are represented in magnitude and direction by two sides of a triangle taken in order then their third side will represent the resultant of two forces in the direction and magnitude taken in opposite order.

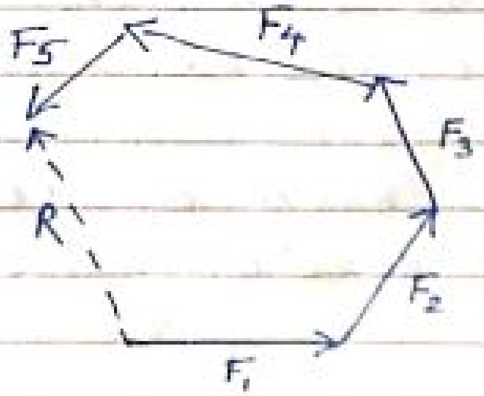




Polygon Law of Forces

This law is basically the extension of the traingular law of forces. If a material body is subjected to more than 3 coplanar, non-collinear, concurrent forces, then polygon law is used to find the resultant force.





The Polygon law of forces states that:

If three or more coplanar, non ~~coplanar~~ collinear, concurrent forces constitute the force system and can be represented in magnitude and direction by the sides of a polygon, taken in order, then one side will remain open.

This side of the polygon, if joined properly and in reverse order, it will give the resultant of the force system. If the ~~material~~ material body remains in equilibrium under the action of such type of force system, then a closed polygon is obtained.

Equilibrium of Forces

If a body is moving at constant velocity or the body is at rest then the body is said to be in equilibrium in a state

If a number of forces are acting on the body and its resultant comes out to be zero, then the body is said to be in equilibrium.

* Such a set of forces, whose resultant is zero are called equilibrium forces.

Equilibrium of Forces

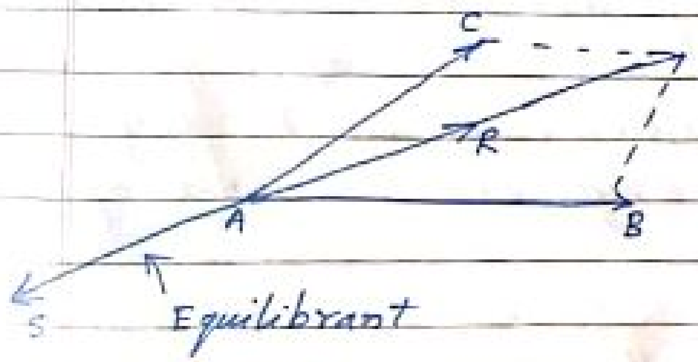
Equilibrium of ~~collinear Forces~~ concurrent coplanar Forces

When a body is acting upon by number of concurrent forces, the combined effect ~~in~~ is equivalent (equal) to the effect of the resultant

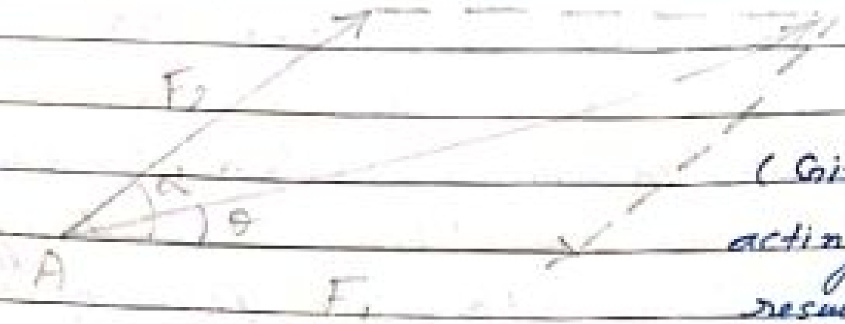
If the effect of the resultant on the body is zero then the combined effect of system of forces on that body is also zero.
At that condition the body is said to be static- equilibrium.

The system of forces, where resultant is zero are called equilibrium forces and the force which brings the effect of the resultant into zero is called equilibrant.

Example of Equilibrant



The law of equilibrium can be applied to the two concurrent non-collinear forces acting at a point, it ~~meanc~~ means it holds the two forces in equilibrium by applying at a point A.



(Given forces $\vec{F_1}$ & $\vec{F_2}$ acting at A & their resultant R)

- In the above figure, a force equal and opposite to their resultant, this force is called the equilibriant of the two given forces.

- The forces F_1 & F_2 acting at A have their resultant R, which gives as the diagonal of the parallelogram drawn on side F_1 & F_2.

Equilibrium of collinear Forces

From the principle of parallelogram of forces it follows that the two forces applied at a point can always be replaced by their resultant which is equivalent to the applied forces.

As a result that two concurrent can be in equilibrium only if their resultant force is zero. This is called equilibrium forces, which means that both the two forces are equal in magnitude and opposite in direction

Equilibrium law of collinear forces

Two forces acting at a point can be in equilibrium only if they are equal in magnitude and opposite in direction and collinear in action.

Principles of Equilibrium

Three important principles of equilibrium are:

1) Two Force principle : If a body in equilibrium is acted upon by two forces, then they must be equal, opposite and collinear.

2) Three force principal : If body in equilibrium is acted upon by three forces, then the resultant of any two forces must be equal, opposite and collinear with the third

3) Four force principle : If a body in equilibrium is acted upon by four forces, then the resultant of any two forces must be equal, opposite and collinear with the resultant of the other two forces.

Method of equilibrium of forces

① Analitical Methods

② Graphical Methods

Analytical Methods

(i) Lami's theorem

(ii) Method of resolution

(iii) Method of Moment

(iv) Method of virtual work

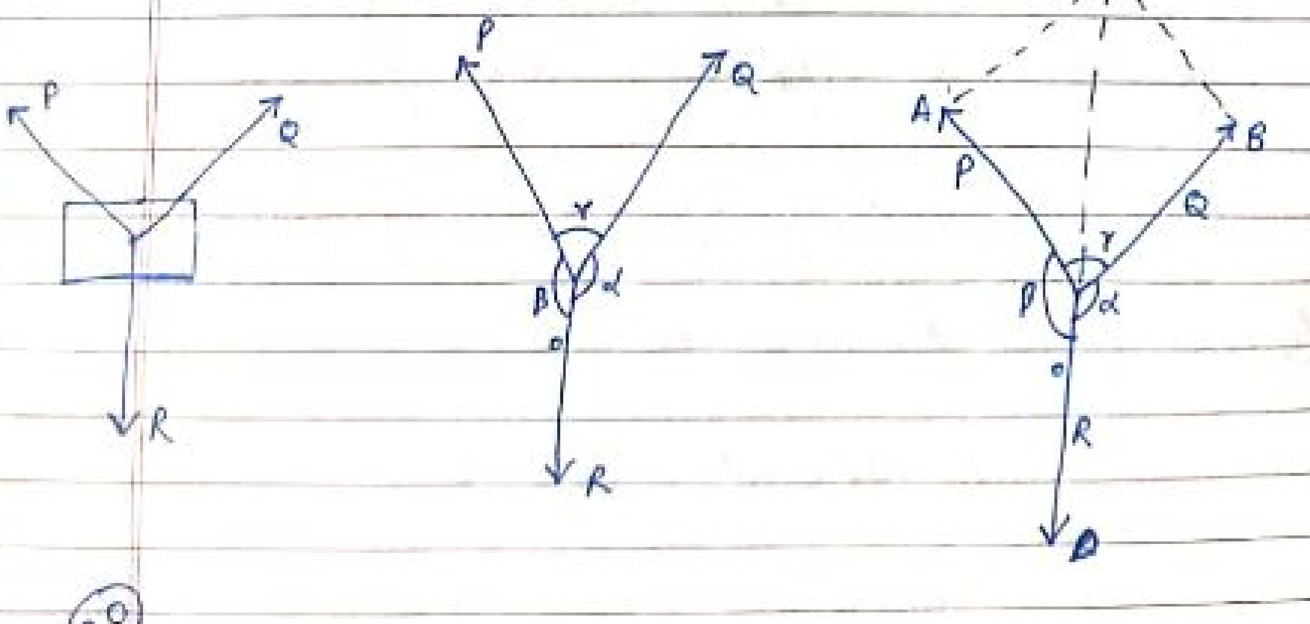
Graphical Method

(i) Triangle law of forces for Equilibrium.

(ii) Polygon law of forces for Equilibrium.

Lami's Theorem

If three coplanar forces acting at a point be in equilibrium, then each force is proportional to the sine of the angle between the other two.



Mathematically,

$$\frac{P}{\sin\alpha} = \frac{Q}{\sin\beta} = \frac{R}{\sin\gamma}$$

Where P, Q and R are three forces and α, β, γ are the angles.

Proof of Lami's Theorm

Consider three co-planer forces P, Q and R acting at a point O. Now complete the parallelogram OACB with OA and OB as adjacent sides.
The resultant of two forces P and Q is diagonal OC both in magnitude and direction of the parallelogram OACB.

Since these forces are in equilibrium, therefore the resultant of the forces P and Q must be in line with OD and equal to R, but in opposite direction.
From the geometry of the figure,

BC = P and AC = Q

$\angle AOC = (180° - \beta)$
$\angle ACO = \angle BOC = (180° - \alpha)$

$\angle CAO = 180° - (\angle AOC + \angle ACO)$
$\Rightarrow \angle CAO = 180° - [(180° - \beta) + (180° - \alpha)]$
$\Rightarrow \angle CAO = 180° - 180° + \beta - 180° + \alpha$
$\Rightarrow \angle CAO = \alpha + \beta - 180°$ ——— (i)

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We know: $\alpha + \beta + \gamma = 360^\circ$

So, $\alpha + \beta + \gamma - 180^\circ = 360^\circ - 180^\circ$

$\Rightarrow (\alpha + \beta - 180^\circ) + \gamma = 180^\circ$ —— (ii)

From eq (i) & (ii) we get

$\angle CAO = 180^\circ - \gamma$

We know that in triangle AOC

$$\frac{OA}{\sin \angle ACO} = \frac{AC}{\sin \angle AOC} = \frac{OC}{\sin \angle CAO}$$

$$\Rightarrow \frac{OA}{\sin(180^\circ - \alpha)} = \frac{AC}{\sin(180^\circ - \beta)} = \frac{OC}{\sin(180^\circ - \gamma)}$$

$$\Rightarrow \frac{P}{\sin \alpha} = \frac{Q}{\sin \beta} = \frac{R}{\sin \gamma}$$

Method of Resolution

This method is applicable to equilibrium of any number of concurrent forces.

From the method of resolution, the resultant force R can be

$$R^2 = \Sigma x^2 + \Sigma y^2$$

$$\boxed{R = \sqrt{\Sigma x^2 + \Sigma y^2}}$$

But for equilibrium $R = 0$, hence $\Sigma X = 0$ and $\Sigma Y = 0$. ie, the sum of ~~rev~~ resolution part of all part of all forces in x axis & y axis are zero.

Method of Moment

The algebric sum of moments of forces about any point is zero.

ie $\boxed{\Sigma M = 0}$

Method of virtual work

The algebric sum of virtual work is zero.

ie $\boxed{\Sigma W = 0}$

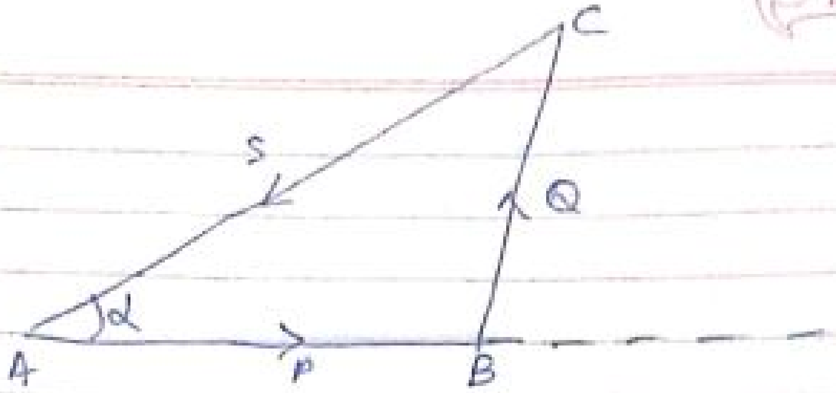
where (virtual work = Force × virtual ~~displacmt~~ displacement.

Graphical Method

Triangle law of forces for Equilibrium

If the body is in equilibrium under the action of three forces, then they must form a closed traingle of same order

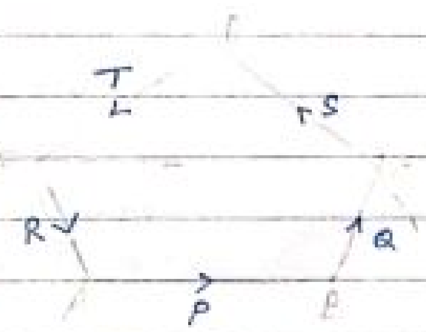
The forces are drawn to the scale and any unknown force can be determined by measuring the length of the side of that triangle.



Polygon Law of forces for Equilibrium

If the body in equilibrium under the action of number of forces then they must form a closed polygon of same order.

The forces are drawn to the choosen scale and one or two forces can be determined by measuring the length of the sides of the polygon.



Condition of Equilibrium

i) A body is said to be in equilibrium under the action of coplaner concurrent force, if

$$\boxed{\Sigma x = 0, \ \Sigma y = 0}$$

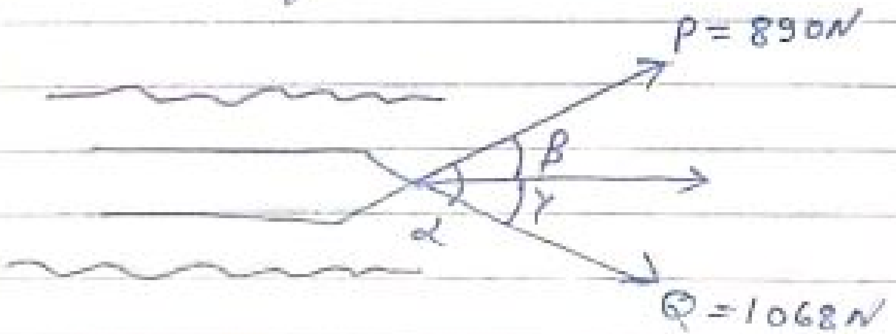
v) A body is said to be in equilibrium under the action of any coplaner system of forces, if

$$\boxed{\Sigma X = 0, \ \Sigma Y = 0, \ \Sigma M = 0}$$

Numericals of Coplaner forces

1) A boat is moved uniformly along a canal by two horses pulling with forces P = 890 N and Q = 1068 N acting under an angle $\alpha = 60°$, as shown in fig.

Determine the magnitude of the resultant pull on the boat and the angles β and γ as shown in fig.



Solⁿ:

$P = 890\,N, \quad Q = 1068\,N \quad \alpha = 60°$

From Parallelogram law:

$$R = \sqrt{P^2 + Q^2 + 2PQ \cos\alpha}$$

$$= \sqrt{(890)^2 + (1068)^2 + 2 \times 890 \times 1068 \times \cos 60°}$$

$$= 1698.01\,N \quad \text{(Ans)}$$

The direction of the resultant R with respect to Q :

$$\tan \gamma = \frac{P \sin \alpha}{Q + P \cos \alpha}$$

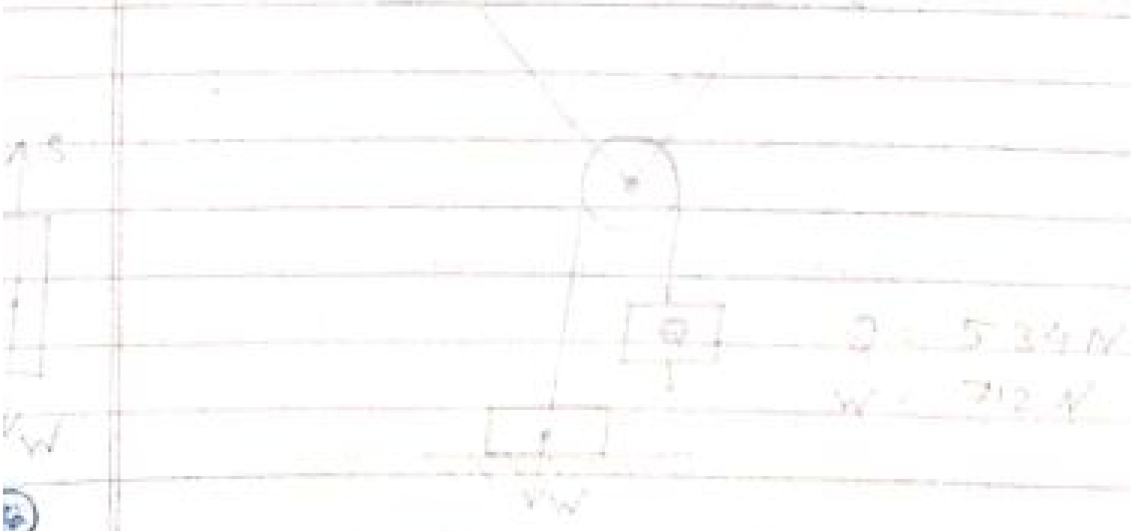
$$= \frac{890 \sin 60^\circ}{(1068 + 890 \cos 60^\circ)}$$

$$= 0.5094$$

or $\gamma = \tan^{-1}(0.5094) = 26^\circ 59'$

Now, $\beta = \alpha - \gamma = 60^\circ - 26^\circ 59' = 33^\circ 1'$ (Ans)

2) A man of weight $W = 712\,N$ holds one end of a rope that passes over a pulley vertically above his head and to the other end of which is attached a weight $Q = 534\,N$. Find the force with which the man's feet press against the floor.



Sol^n: $W = 712\ N$, $Q = 534\ N$

The tension in the rope is equal to the load hanged at the free end. Hence, we have, $T = Q$. The angle between T and W is $180°$. Applying law of parallelogram the resultant

$$R = \sqrt{W^2 + S^2 + 2WS\cos 180°} \qquad \because W = S$$

$$= \sqrt{W^2 + S^2 \mp 2WS}$$

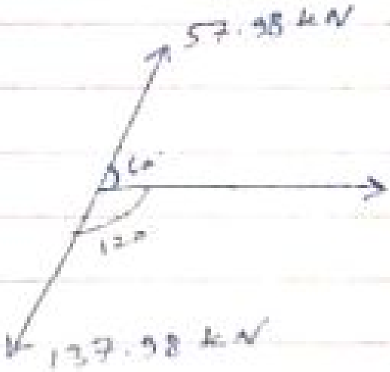
$$= \sqrt{(W-S)^2}$$

$$= W - S$$

$\therefore$ $R = (712 - 534)\ N = 178\ N\ (\downarrow)$

3) In a concurrent system of forces, two forces P and Q act on a point at an angle of $60°$. The resultant force R is 120 kN and P is 80 kN. Determine the value of Q.

Sol Given, $R = 120\ kN$, $P = 80\ kN$ $\theta = 60°$



$R = \sqrt{P^2 + Q^2 + 2PQ\cos\theta}$

$\Rightarrow R^2 = P^2 + Q^2 + 2PQ\cos\theta$

$\Rightarrow 120^2 = 80^2 + Q^2 + 2 \times 80 \times Q \cos 60°$

$\Rightarrow 14400 = 6400 + Q^2 + 80Q$

$\Rightarrow Q^2 + 80Q + 6400 - 14400 = 0$

$\Rightarrow Q^2 + 80Q - 8000 = 0$

$$\Rightarrow Q = \frac{-80 \pm \sqrt{(80)^2 - 4 \times (1) \times (-8000)}}{2 \times 1}$$

$$\Rightarrow Q = \frac{-80 \pm \sqrt{6400 + 32000}}{2} = \frac{-80 \pm 195.96}{2}$$

$$\Rightarrow Q = \frac{-80 + 195.96}{2} \text{ and } \frac{-80 - 195.96}{2}$$

$\Rightarrow Q = 57.98$ kN and -137.98 kN

when Q is −ve, the included angle is 120°. Hence the value of Q = −127.98k is neglected

So, Q = 57.98 kN (Answer)

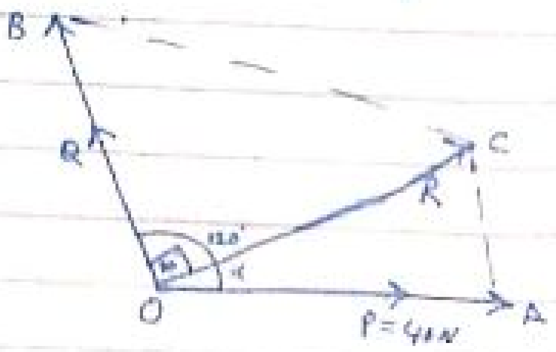
4) Two forces are acting at an angle of 120°. The greater is of 40 N and the resultant is acting at 90° to the smaller force. Find the magnitude of the smaller force.

Sol:

$\theta = 120°$

$P = 40N$

$\alpha = 120° - 90° = 30°$



Now we know: $\boxed{\tan\alpha = \dfrac{Q\sin\theta}{P+Q\cos\theta}}$

$$\Rightarrow \frac{Q\sin 120^\circ}{40 + Q\cos 120^\circ} \Rightarrow \frac{Q \times 0.866}{40 + Q(-0.5)}$$

$$\Rightarrow 40 - 0.5Q = 1.5Q$$

$$\Rightarrow 40 = 2Q$$

$$\Rightarrow Q = 20\ N \quad Ans$$

* Analytical determination of resultant of a number of concurrent forces:

$$\text{Resultant } (R) = \sqrt{(\Sigma H)^2 + (\Sigma V)^2}$$

$$\text{or} \quad (R) = \sqrt{(\Sigma F_x)^2 + (\Sigma F_y)^2}$$

$$\tan\theta = \frac{\Sigma V}{\Sigma H} \text{ or } \frac{\Sigma F_y}{\Sigma F_x}$$

Q1> A body is acted upon by three forces 2kN, 4kN and 1kN. Force 2kN is horizontal and acts towards right force 4kN acts at 45° to the horizontal and is inclined right upward and the force 1kN is vertical. Determine the resultant of three forces

Soln: ΣH = Algebric sum of the resolution parts of the given forces along horizontal direction.

$$\Sigma H = 2\cos 0^\circ + 4\cos 45^\circ + 1\cos 90^\circ$$

$= 2 + 4 \times \frac{1}{\sqrt{2}} + 0 = 2 + 2\sqrt{2} = 2 + 2 \times 1.414$

$= 4.828 \text{ kN}$

ΣV = algebric sum of the resolved parts of the given forces along vertical direction

$\Sigma V = 2 \sin 0° + 4 \sin 45° + 1 \sin 90°$

$\Sigma V = 0 + 4 \times \frac{1}{\sqrt{2}} + 1$

$= 2\sqrt{2} + 2$

$= 3.828 \text{ kN}$

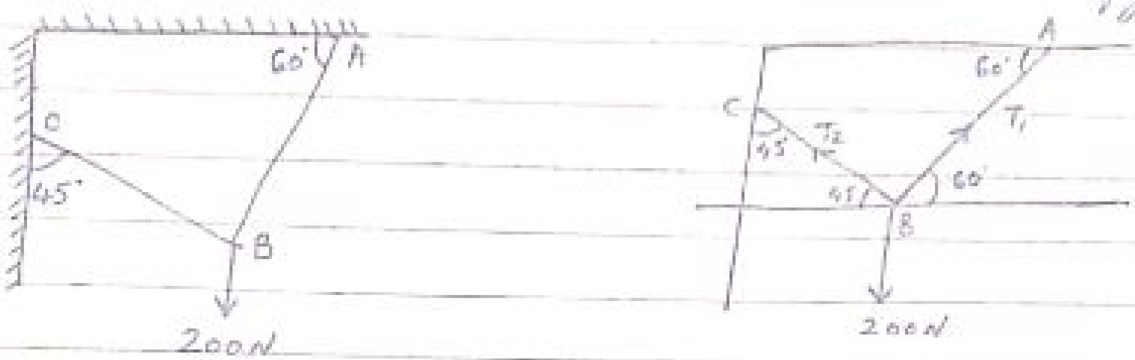
$\therefore \quad R = \sqrt{(\Sigma H)^2 + (\Sigma V)^2} = \sqrt{(4.828)^2 + (3.828)^2}$

$= 6.1614 \text{ kN}$

$\tan \theta = \frac{\Sigma V}{\Sigma H} = \frac{3.828}{4.828}$

$\Rightarrow \theta = 38.41°$

1) An electric light fixture of weight 200N is supported as shown in the figure. Determine the tensile forces in the wires BA and BC as shown in the figure



let, T_1 = Tension in wire BA

T_2 = Tension in wire ~~EB~~ BC

Now, Point B is in the equilibrium under the action of the following forces

(a) T_1 = Tension in wire BA

(b) T_2 = Tension in wire BC

(c) Weight of light fixture, 200 N

Sol^n: Applying Lami's theorem to point B

$$\frac{T_1}{\sin(90°+45°)} = \frac{T_2}{\sin(90°+60°)} = \frac{200}{\sin \angle ABC}$$

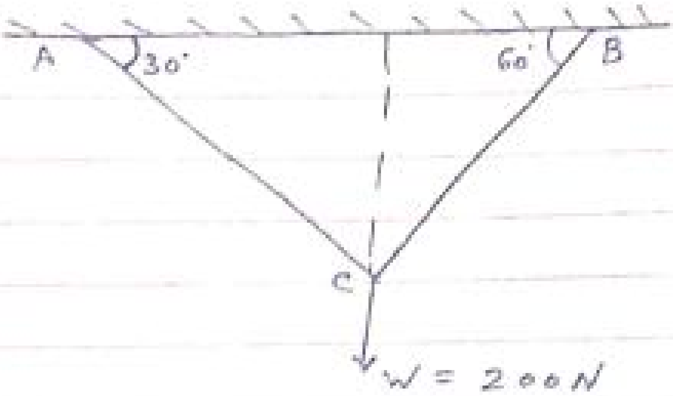
$$\Rightarrow \frac{T_1}{\sin 135} = \frac{T_2}{\sin 150°} = \frac{200}{\sin 75}$$

$$\Rightarrow \frac{T_1}{0.707} = \frac{T_2}{0.5} = \frac{200}{0.965}$$

$$\Rightarrow T_1 = 0.707 \times \frac{200}{0.965} = 146.5\text{ N}$$

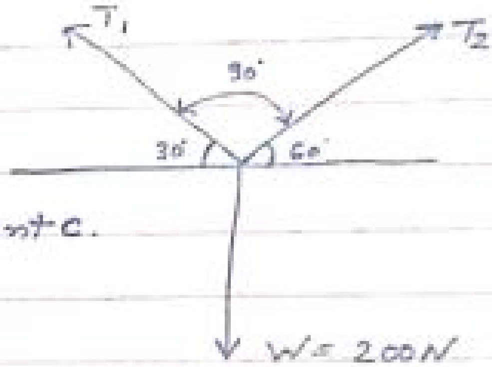
$$T_2 = 0.5 \times \frac{200}{0.965} = \cancel{10.36\,N}\ 103.6\,N$$

2) A weight of 200N is supported by two chains AC and BC as shown in the figure. Find out the tension in each chain.



Sol^n

Let T_1 = Tension in chain AC
T_2 = Tension in chain BC



Applying lamis thearm to point c.

$$\Rightarrow \frac{T_1}{\sin 150^\circ} = \frac{T_2}{\sin 120^\circ} = \frac{W}{\sin 90}$$

$$\Rightarrow \frac{T_1}{0.5} = \frac{T_2}{0.866} = \frac{200}{1}$$

$$\therefore \frac{T_1}{0.5} = \frac{200}{1} \Rightarrow T_1 = 200 \times 0.5 = 100N$$

$$\therefore \frac{T_2}{0.866} = \frac{200}{1} \Rightarrow T_2 = 200 \times 0.866 = 173.2N$$

Coplanar Non-concurrent Forces

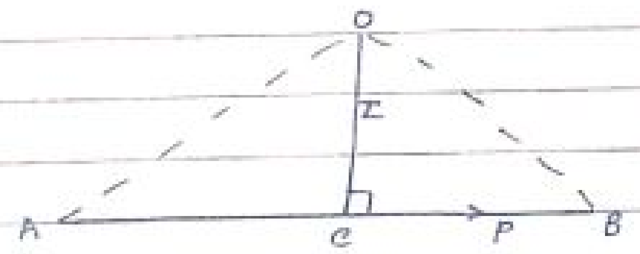
A co-planar non-concurrent system of forces consists of forces which lie in the same plane but line of action of all the forces do not pass through a single point.

Moment of a Force

Moment of a force about a point may be defined as the turning effect of the force about that point.

* Moment of the force is expressed as the product of the force and the perpendicular distance of the point, about which the moment is to be found out and the line of the action of the force.

Graphical representation of Moment



Let Force 'P' be represented in magnitude and direction by the vector AB and 'O' is the point about which the moment is to be found out. Then draw a perpendicular from O onto AB at C.

Now, $OC = I$

Point 'O' is known as "moment of ~~a~~ force centre" and the distance OC = l is known as "the moment arm" or "arm of the force". Join AO and BO to complete the traingle AOB. Moment of Force P about 'O'.

$$= P \times OC = AB \times OC$$
$$= 2 \times \text{Area of triangle } AOB$$

* * Now, we may conclude that the moment of a force about any point is equal to twice the area of the triangle, whose base represents the force to some scale and whose vertex is the point about which the moment is taken.

unit of Moment

unit of moment a force about a point depends upon the unit of force and unit of distance.

Force is measured in Newton (N) and distance is measured in Meter (m). So the moment will be expressed in Newton - meter. (Nm)

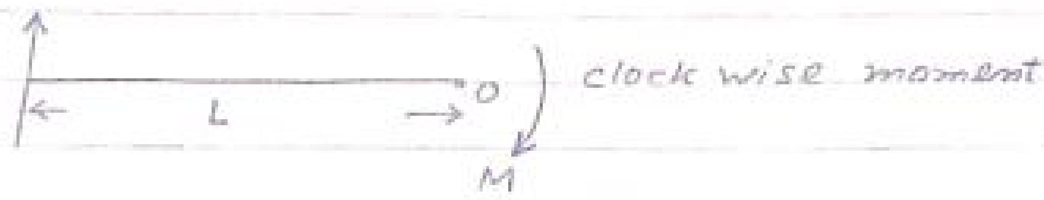
So, unit of moment is Newton-meter (Nm).

Types of Moments

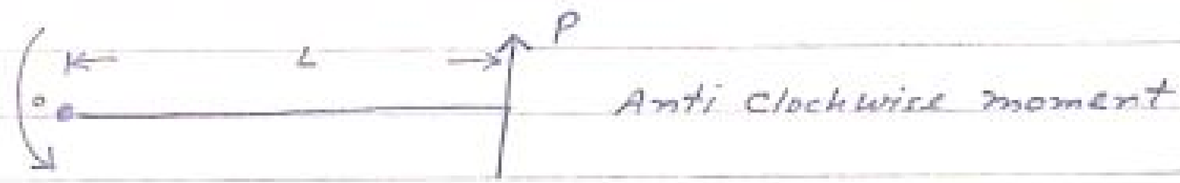
Moments are classified in two types

a) Clockwise Moment b) Anti clock wise Moment

A clockwise moment turns or rotates the body about the point in the same direction in which hands of a clock moves.



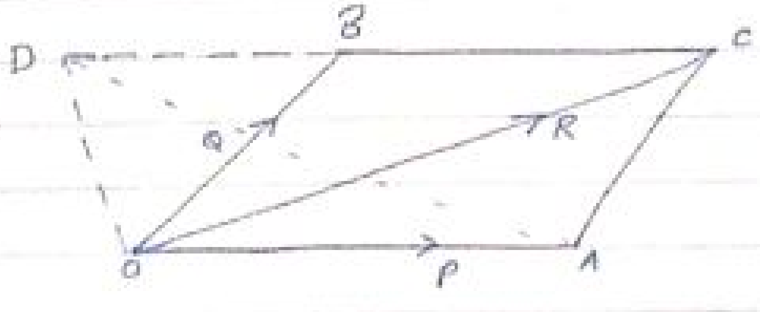
An anti clockwise moment turns or rotates the body about the point in opposite direction in which the hands of a clock moves.



Varignon's Theorem

Statement: The algebric sum of the moments of a number of co-planer forces about any point in their plane is equal to the moment of their resultant about the same point.

- When all the forces are concurrent



Let P and Q be two co-planar forces acting at point O as shown in fig above. Let D be any point in their plane.
Draw DC parallel to OA to meet OB at B. OA and OB represent the forces P and Q respectively, in magnitude and direction. Complete the parallelogram OABC. Join OD & AD. OC represents the resultant R ~~and~~ of P and Q in magnitude and direction.

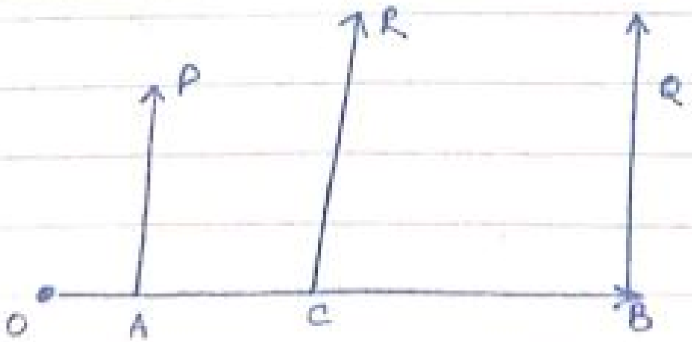
Now, Moments of P, Q and R about D are given by $2\Delta AOD$, $2\Delta BOD$ and $2\Delta COD$.
We have moment of resultant R about

$$D = 2\Delta COD$$
$$= 2[\Delta BOD + \Delta BOC]$$
$$= \cancel{2[} 2\Delta BOC + 2\Delta AOC \cancel{=} 2\Delta BOD + 2\Delta AOD$$
$$= 2\Delta BOD + 2\Delta AOD$$

= (Moment of Q about D) + (Moment of P about D)

Triangles AOC and AOD are equal in area as they are on the same base OA and between the same lines OA and CD.

- when all the forces are parallel



Consider two like parallel forces P and Q acting in the same and O be any point in their plane.

Let, R be the resultant of P and Q

Now,

$$R = P + Q$$

Draw the line OACB perpendicular to all the forces P, Q and R intersecting them at A & B and C respectively.

The algebraic sum of the moments of P and Q about O.

$$= -(P \times OA) - (Q \times OB)$$

$$= -[P(OC - AC)] - [Q(OC + BC)]$$

$$= -(P \times OC) + (P \times AC) - (Q \times OC) - (Q \times BC)$$

As, $P \times AC = Q \times BC$. For two like parallel forces P and Q, $P + Q = R$ and R acts at C in between A and B such that $P \times AC = Q \times BC$

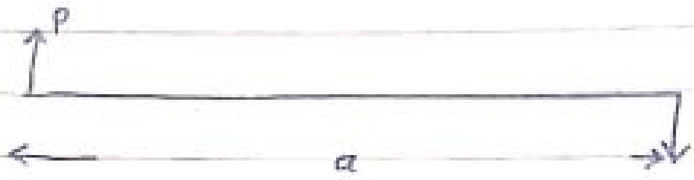
$$= -[(P \times OC) + (Q \times OC)]$$

$$= -[(P+Q) \times OC] = -(R \times OC)$$

= Moment of R about O.

Couple

A couple is defined as a pair of two equal and unliked parallel forces, separated by a finite distance.



The perpendicular distance between the lines of action of the forces of the couple is called the arm of couple.

The product of either force of couple with the arm of the couple is called the moment of the couple. The rotational effect of a couple is measured by its moment.

$\therefore$

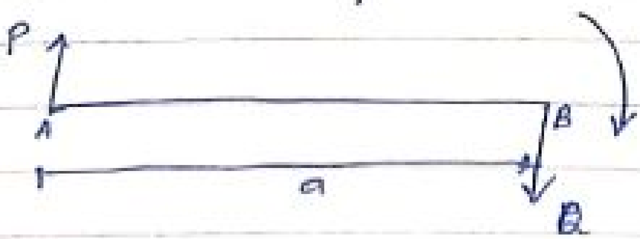
$M = P \times a$ where: P = magnitude of Force.

a = arm of the couple

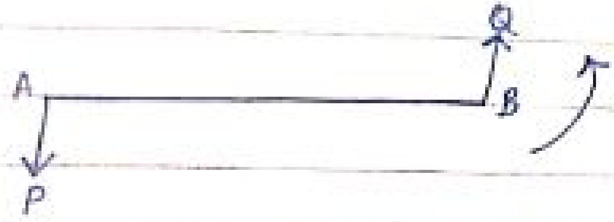
Classification of Couples

Depending on the direction in which a couple tends to rotate the body on which it acts, the couples may be classified as

a) Clockwise Couple



(b) Anti Clock wise Couple



Characteristics of a couple

a) The algebraic sum of the forces, constituting the couple is zero.

b) The algebraic sum of the moments of two forces forming a couple about any point in their plane is constant and equal to the moment of the couple.

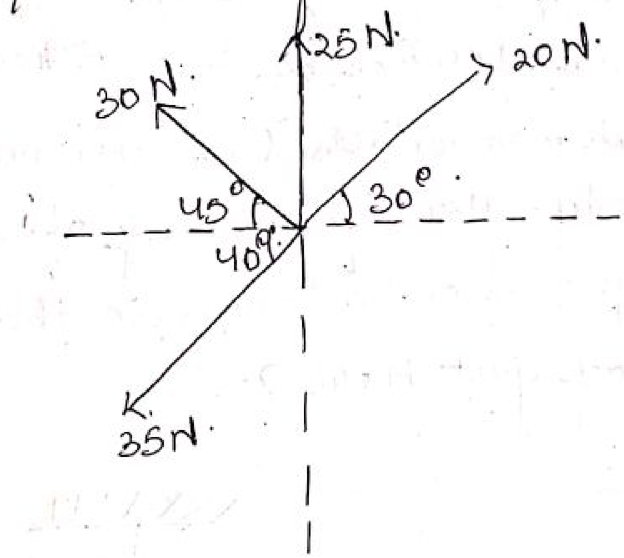
c) Two coplanar couples, whose moments are equal and opposite balance each other.

d) Any two couples, whose moments are equal and of the same sign, and are equivalent both in magnitude and direction.

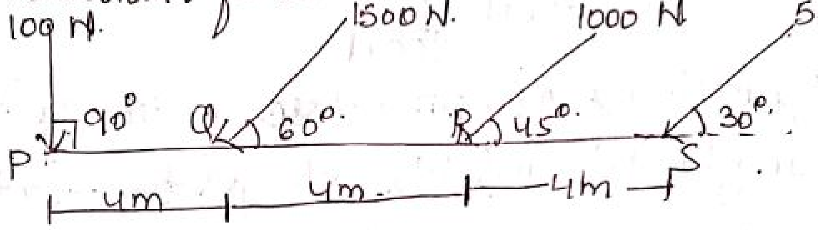
e) Any number of co-planar couples can be reduced to a single couple, whose magnitude will be equal to the algebraic sum of all the couples.

Questions

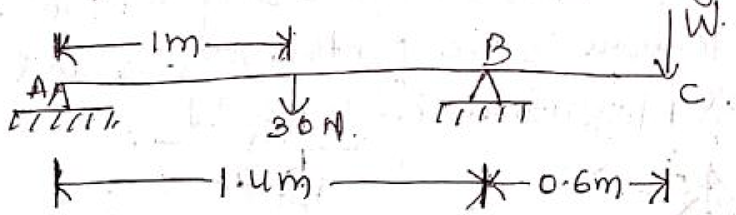
Q. Find the resultant force.



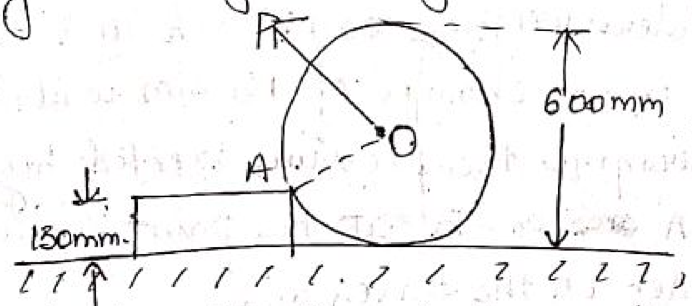
Q. Find the magnitude, direction and the position of the resultant force.



Q. A uniform plank ABC of weight 30 N and 2m long is supported at one end A and at a point 'B', 1.4m from 'A' as shown in the figure.

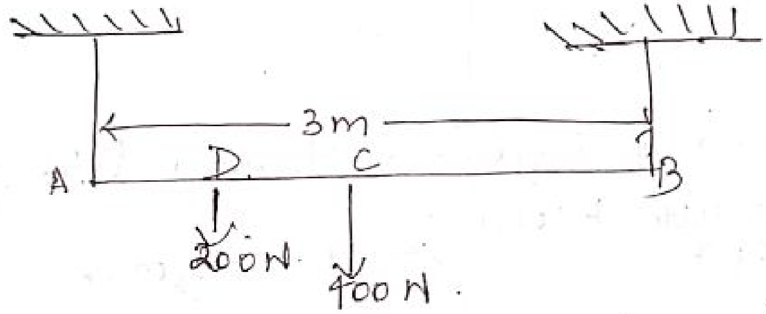


Q. A uniform wheel of 600 mm diameter, weighing 5KN rests against a rigid rectangular block of 150 mm height.

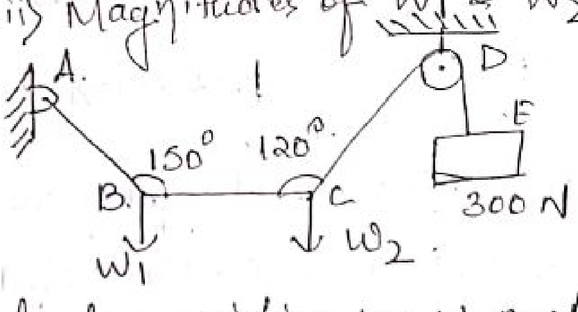


Find the least pull, through the centre of the wheel, required just to turn the wheel over the corner 'A' of the block. Take all the surfaces to be smooth.

Q. A beam 3m long weighing 400 N is suspended in a horizontal position by two vertical strings, each of which can withstand a maximum tension of 350 N only. How far a body of 200N weight be placed on the beam, so that one the strings may just break?



Q. A light string ABCDE whose extremity 'A' is fixed, has weights W_1 and W_2 attached to it at B and C. It passes round a small smooth peg at 'D'. carrying a weight of 300N at the free end 'E' as shown in the figure. If in the equilibrium position, BC is horizontal and, AB and CD make 150° & 120° with BC; find i) Tensions in the portion AB, BC and CD of the string ii) Magnitudes of W_1 & W_2.

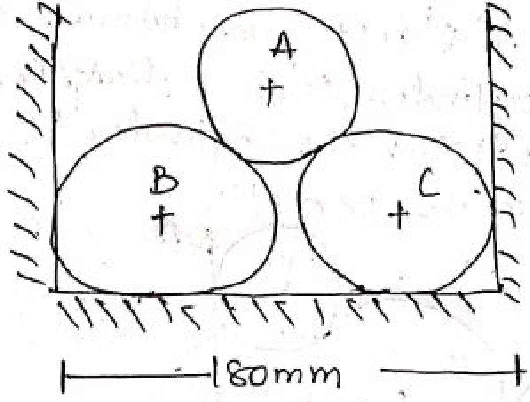


Q. Three cylinders weighing 100 N each and of 80mm diameter are placed in a channel of 180mm width as shown in the figure. Determine the pressure exerted by

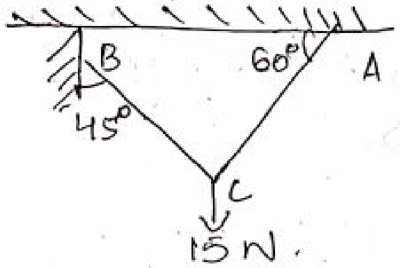
(i) Cylinder 'A' on 'B' at the point of contact.

(ii) The cylinder on the base, and

(iii) The cylinder 'B' on the wall.

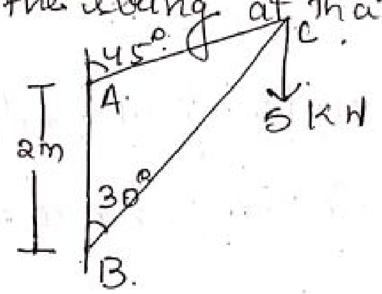


Q. An electric light fixture weighing 15 N hangs from a point C, by two strings AC and BC. The string AC is inclined at 60° to the horizontal and BC at 45° to the horizontal as shown below.

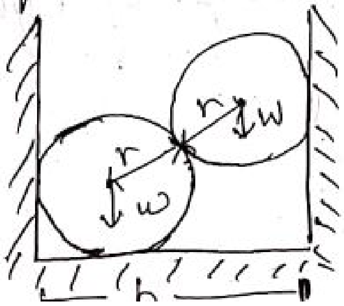


Using Lami's theorem, determine the forces in the strings AC & BC.

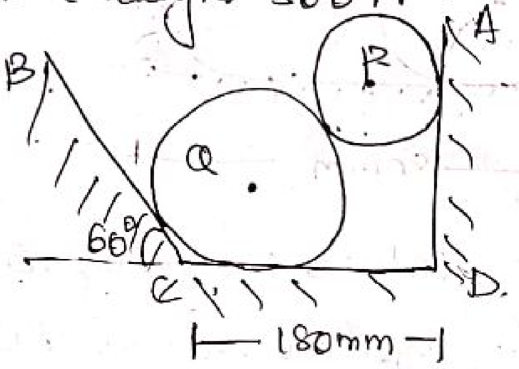
Q. A spherical ball of weight 50N is suspended vertically by a string 50mm long. Find the magnitude and direction of the least force, which can hold the ball. 100mm above the lowest point. Also find the Tension in the string at that point.



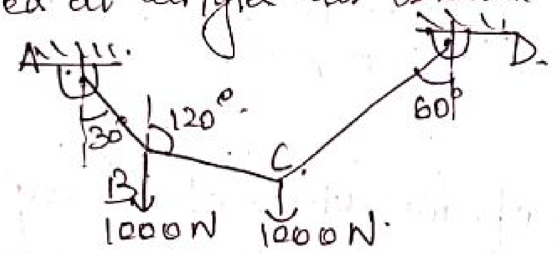
Q. Two smooth spheres of weight 'W' and radius 'r' each are in equilibrium in a horizontal channel of 'A' & 'B' vertical sides as shown in the figure. Find the force exerted by each sphere on the other. Calculate these values, if r = 250mm, b = 900mm and W = 100 N.



Q. Two cylinders 'P' & 'Q' rest in a channel as shown in the figure. The cylinder 'P' has diameter of 100 m & weight 200 N, whereas the cylinder 'Q' has diameter of 180 mm & weight 500 N.



Q. A string ABCD, attached to fixed points A & D has two equal weights of 1000 N attached to it at B & C. The weights rest with the portions AB & CD inclined at angles as shown in the figure.



Q. Define the term, force and state its effect.

Q. State triangle law of forces and polygon law of forces

Q. State the Varignon's principle of moments.

Q. What is a couple? State the characteristics of a couple.

Q. State Lami's theorem.

Q. What is principle of transmissibility?

Q. What are the equations of equilibrium of a body?

Example of couple

a) Winding of a clock
b) Unscrewing the cap of a bottle
c) Locking or unlocking of a lock with a key.
d) Opening or closing of a water tap.

Friction

When a body is made to slide over another body, the stationary body offers force of resistance to the motion of the sliding body over it. This force of resistance is known as the force of friction.

Its presence cause loss of energy, wearing of parts and there by increase losses.

However friction is desired in many devices such as breaks and clutches, belt and rope drives, fastening devices etc for their smooth operation.

Friction is of two types

- Static Friction :
 It is the friction experienced by a body when it is at rest.
 When one body is made to slide over another body and as a result, the body does not move, we can conclude that the frictional

force developed at common surface of contact is equal to the external force, applied for motion.

Dynamic Friction:

It is the friction experienced by a body when it is in motion. It is also called Kinetic friction.

Dynamic Friction is of two types:

(i) Sliding Friction (ii) Rolling Friction

Sliding Friction

When a body is made to slide over another body, then a frictional force acts at the common surface of contact in a direction opposite to the motion. This frictional force is known as Sliding friction.

Rolling Friction

It is the frictional force which opposes the rolling of one body over another.

Limiting Friction

The limiting friction is the maximum value upto which the static friction can be reach and balance the external force applied for motion.

Normal reaction:

A body in its equilibrium position, lying on a horizontal or an inclined surface, exerts a force equal to its own weight on the Surface in a direction vertically downwards through its center of gravity.

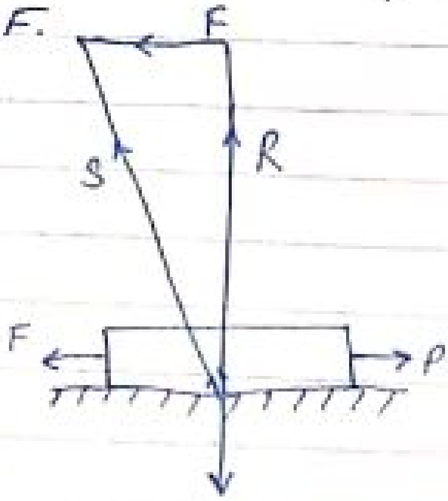
The surface, in turn exerts an upward reaction on the body. The reaction acts perpendicular to the plane and is called normal reaction (R)

The force of friction is directly proportional to normal reaction.

Angle of Friction

It is the angle which the resultant of normal reaction and limiting force and limited force friction.

Let, F = Force of Friction
R = Normal reaction
W = Weight of body
P = External force to move the body
S = Resultunt of R and F.



$$S = \sqrt{R^2 + F^2}$$

$$\tan \phi = \frac{F}{R}$$

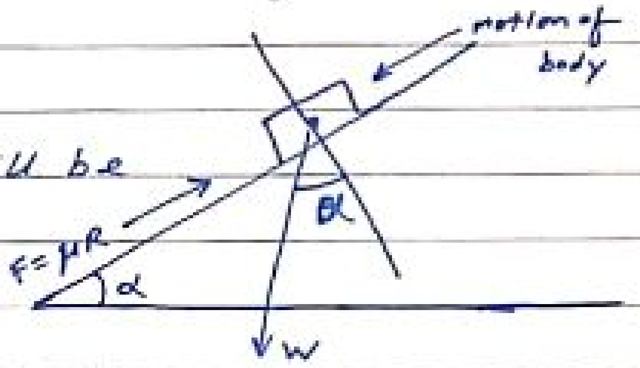
where ϕ = angle of friction

The ratio of force of friction (F) to normal reaction (R) is called coefficent of friction (μ).

$$\mu = \frac{F}{R} = \tan\theta$$

Angle of Repose

If α is gradually increased, then a point will reach where the body will be just at the point of sliding.



At this point, the body will be under the action of the following forces:

i) Weight (W) of the body
ii) Normal reaction (R)
iii) Limiting force is friction $F = \mu R$

$\because$ α = angle, which inclined plane make with the horizontal

Resolving there forces along and perpendicular to the plane, we get

$$F = \mu R = W\sin\alpha \quad \text{—— (i)}$$
$$R = W\cos\alpha \quad \text{—— (ii)}$$

Now, dividing eq (i) by eq (ii)

$$\frac{F}{R} = \frac{W\sin\alpha}{W\cos\alpha} = \frac{\mu R}{R} = \frac{W\sin\alpha}{W\cos\alpha}$$

$$\Rightarrow \mu = \tan\alpha \Rightarrow \tan\phi = \tan\alpha \Rightarrow \phi = \alpha$$

Hence, the angle of inclination (α) of the plane at which the body resting on it, is on the verge of sliding down the plane, is called the angle of repose. It is equal to the angle of friction between the body and the plane.

Laws of Static Friction

1) The Force of friction always acts in a direction opposite to the direction of motion between two Surfaces

2) Force of friction acts along the common plane of contact between two bodies.

3) The force of friction is equal to the external force applied for motion

4) Force of Friction is self adjusting till the body is at rest.

5) The magnitude of limiting friction bears a constant ratio to the normal reaction.

6) Force of friction is independent of the area and shape of the surfaces of contact subject to condition that the normal reaction does not change.

7) Force of friction depends upon the materials of the bodies, in contact with each other and the roughness.

Laws of dynamic Friction

a) The force of friction acts in a direction opposite to the direction of motion between two bodies.

b) The magnitude of dynamic friction bears a constant ratio to normal reaction between the two surfaces

c) The force of friction decrease with increase of motion.

Advantages of Friction

Friction plays a vital role in our daily life. Without friction we are handicap.

1) It becomes difficult to walk on a slippery road due to low friction. So we can only walk on a surface due to friction

2) We can not fix nail in the wood or wall if there is no friction. It is friction which holds the nail.

3) Writing on a paper with pen or pencil is possible due to friction.

Disadvantages of Friction

1) The main disadvantage of friction is that it produces heat in various parts of machines. In this way some useful energy is wasted as heat energy.

2) Due to friction we have to exert more power in machines.

3) It opposes the motion.

4) Due to friction, noise is also produced in machines.

5) Due to friction, engines of automobiles consume more fuel which is a money loss.

Methods of Reducing Friction

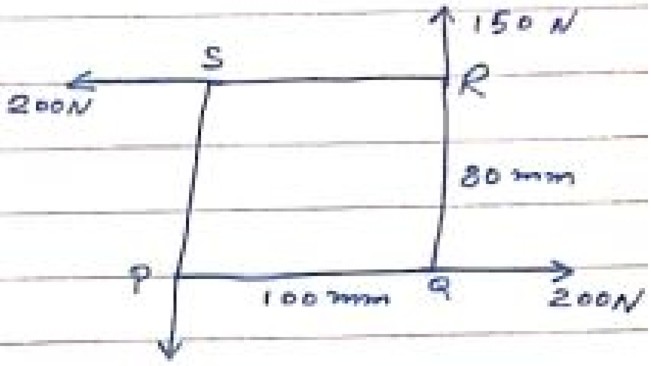
1) use of Grease

2) use of Ball bearing

3) Design Modification

Problems of couples

Q) A rectangle PQRS has sides PQ = RS = 100 mm and QR = SP = 80 mm. Forces of 200 N each act along PQ and RS and forces of 150 N each act along QR and SP. Find out the resultant moment of the system of forces.



The forces of 200 N each will give rise to couple of moment,

$= 200 \times 80 = 16000$ N-mm

$= 16$ N-m

The forces of 150 N each will give rise to couple of moment,

$= 150 \times 100 = 15000$ N-mm $= 15$ N-m

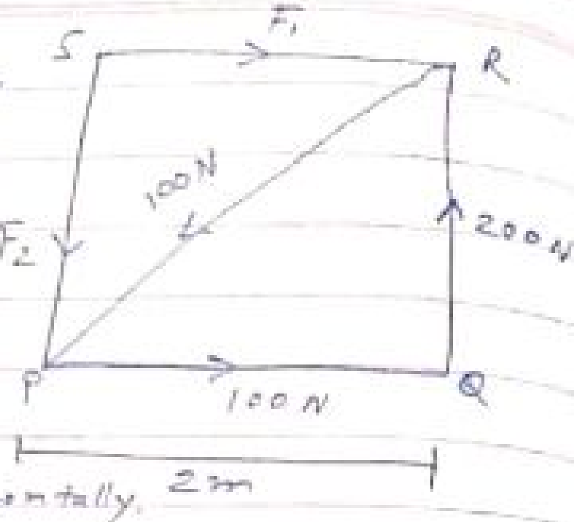
Now, resultant moment of the system

$= 16 + 15 = 31$ Nm

This will be an anti clock wise moment.

Q) A square block PQRS of side 2 m. is acted upon by a system of forces along its sides as shown in the figure. Find the value of F_1 and F_2, if the system reduces to a couple. Find the magnitude of the couple.

Sol: We know that if the system reduces to a couple, the resultant force in ~~horizontal~~ horizontal and vertical directions will be zero.



Now, Resolving the forces horizontally.

$$100 - 100\cos 45^\circ - F_1 = 0$$

$$\Rightarrow F_1 = 100 - 100\cos 45^\circ$$
$$= 29.3\ N$$

Now, Resolving the forces vertically.

$$200 - 100\sin 45^\circ - F_2 = 0$$

$$\Rightarrow F_2 = 200 - 100\sin 45^\circ = 129.3\ N$$

Now, moment of the couple is equal to the algebraic sum of the moments about any point.

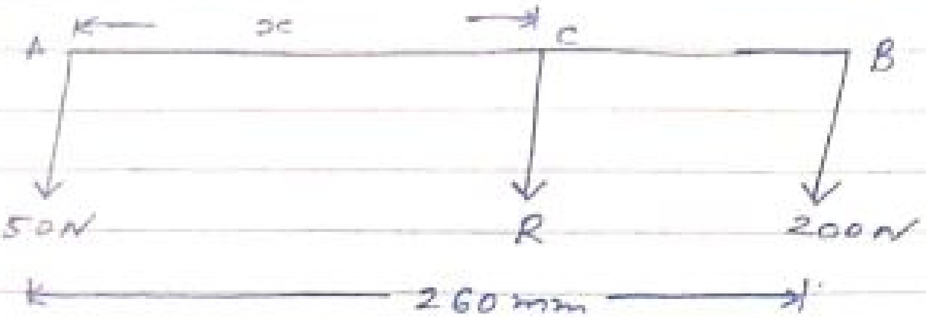
Taking moments of the couple or taking moments about P, we have,

$$(-200\times 2) + (-F_1\times 2) = -400 - 29.3\times 2$$
$$= -400 - 58.6$$
$$= -458.6\ Nm$$

Problems of Parallel Forces

Q) Two like parallel forces of 50N and 200N act at the ends of a rod 260 mm long. Find the magnitude of the resultant force and the point where it's acts.

Sol:



The given forces are like parallel forces.

Hence,

$$R = 50 + 200 = 250\,N$$

Let, AC = distance between the lines of action of the resultant forces R and 50N Force = x.

$$50x = 200\,(260 - x)$$

$$\Rightarrow 50x = 200 \times 260 - 200x$$

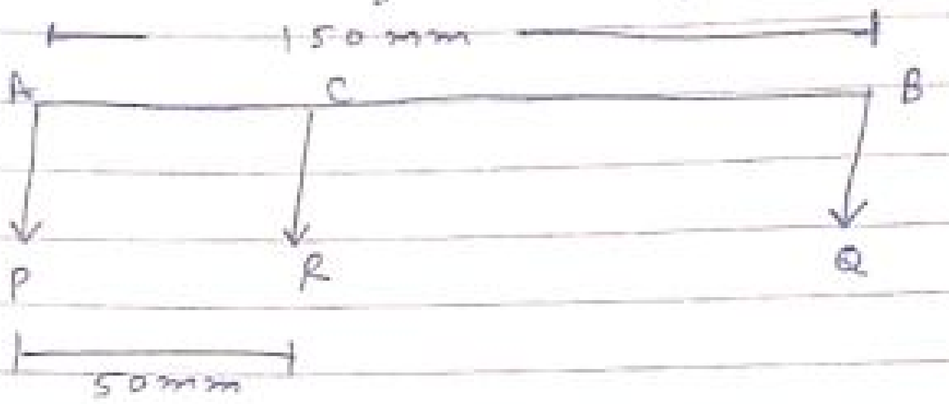
$$\Rightarrow 250x = 52000$$

$$\Rightarrow x = \frac{52000}{250}$$

$$= 208\,m.$$

Q) Two like parallel forces are acting at a distance of 150 mm from each other. These forces are equivalent to a single force of 300N and its line of action is at a distance of 50 mm from one of the forces.

Find out the magnitude of two forces.



Sol: $R = P + Q = 300\,N$

P and Q are two parallel forces acting at A and B. R is the resultant acting at C.

$AB = 150\,mm$

$AC = 50\,mm$

$R = P + Q = 300\,N$ — (i)

Taking moments about C, we get

$P \times AC = Q \times BC$

$\Rightarrow P \times 50 = Q \times 100$

$\Rightarrow P = 2Q$ — (ii)

Now, $P + Q = 300$

$\Rightarrow 2Q + Q = 300$

$\Rightarrow 3Q = 300 \Rightarrow Q = 100\,N$

$P = 2Q = 2 \times 100 = 200\,N$

Problems of Frictions

Q) A body of weight 600 is lying on a horizontal plane. The coefficient of friction is 0.4. Determine the magnitude of the force that will move the body, when applied at an angle of 30° to the horizontal.

Sol: Weight of the body $W = 600\ N$

P = Magnitude of the force applied to move the body.

F = Force of friction

Now resolving the forces horizontally,

$F = P\cos 30° = 0.866\ P$

Now resolving the forces vertically,

$R + P\sin 30 = W$

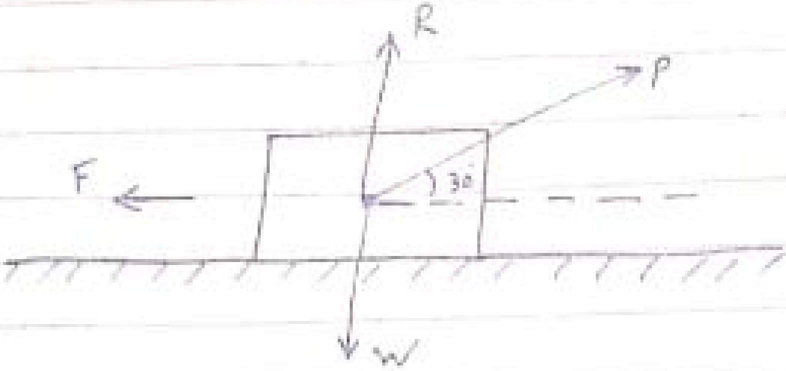
$\Rightarrow R = W - P\sin 30° = 600 - P \times 0.5$

Now, $F = \mu R$

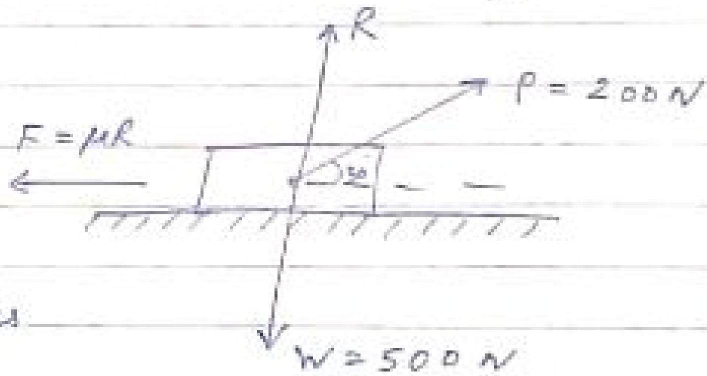
$\Rightarrow 0.866\ P = 0.4\ (600 - P \times 0.5)$

$\Rightarrow 0.866\ P = 240 - 0.2\ P = 1.066P = 240$

Thus, $P = 225.14\ N$



Q) A body of weight 500N is placed on a horizontal plane. A pull of 200 N applied at an angle of 30° with the horizontal just makes the body to slide. Find out normal reaction and coefficient of friction.



Soln:

Resolving the forces ~~hor~~ horizontally,

$F = P\cos 30 \Rightarrow F = 200\cos 30°$

$\Rightarrow F = 200 \times 0.866 = 173.2\,N$

Similarly, resolving the forces vertically

$R + P\sin 30° = W$

$\Rightarrow R = W - P\sin 30 = 500 - 200\sin 30°$

$\Rightarrow R = 500 - 200 \times 0.5$
$= 400N$

$\therefore \quad F = \mu R$

$\Rightarrow \mu = \dfrac{F}{R}$

$\Rightarrow \mu = \dfrac{173.2}{400}$

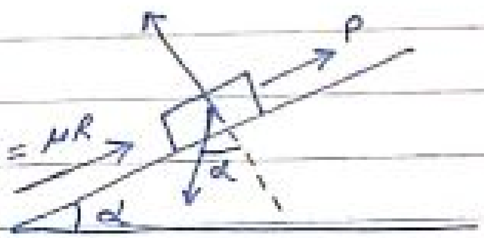
$= 0.433$

Equilibrium of bodies on level plane

Case I: when the force is applied parallel to the plane.

a) when the body is at the point of moving downward.

The applied force is acting



Let: R = reaction of the plane
F = Force of Friction
P = Applied force

Resolving force along the plane,

$P + F = W \sin\alpha$

$\Rightarrow P + \mu R = W \sin\alpha$

$\Rightarrow P = W \sin\alpha - \mu R$ — (eq 1)

Resolving force perpendicular to the plane.

$R = W\cos\alpha$ — (eq 2)

w. Substituting this value of R in eq ②

$$P = W \sin\alpha - \mu R$$
$$= W \sin\alpha - \mu \cdot W\cos\alpha$$
$$= W (\sin\alpha - \mu\cos\alpha)$$
$$= W(\sin\alpha - \tan\phi \cdot \cos\alpha)$$

(where $\mu = \tan\phi$ and ϕ = angle of friction)

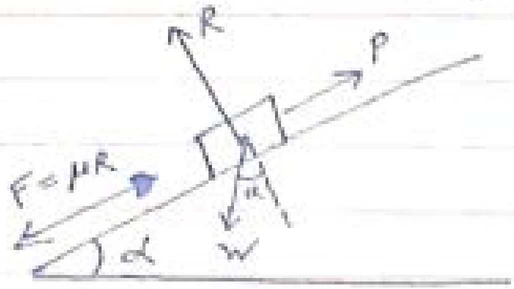
$$P = W\left(\sin\alpha - \frac{\sin\phi \cdot \cos\alpha}{\cos\phi}\right)$$

$$= \frac{W}{\cos\phi}(\sin\alpha \cdot \cos\phi - \sin\phi \cdot \cos\alpha)$$

$$\boxed{P = \frac{W \cdot \sin(\alpha - \phi)}{\cos\phi}}$$

This is the minimum force required to keep the body in equilibrium when the body is at point of sliding downwards.

b) when the body is at the point of moving upwards



Resolving all the forces along the plane

$$P = \mu R + W\sin\alpha \quad \text{---} ③$$

Resolving forces perpendicular to the plane

$$R = W\cos\alpha$$

Putting the value of R in eq ③

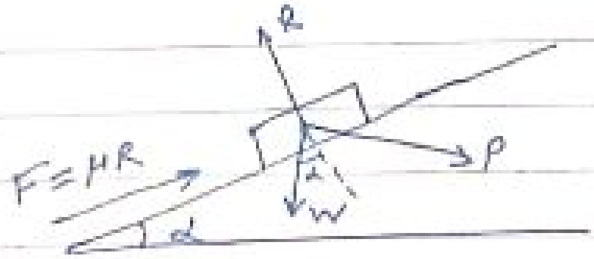
$$P = \mu R + W\sin\alpha$$
$$= \mu . W\cos\alpha + W\sin\alpha$$
$$= \tan\phi . W\cos\alpha + W\sin\alpha$$
$$= W . \frac{\sin\phi}{\cos\phi} . \cos\alpha + W\sin\alpha$$
$$= \frac{W}{\cos\phi}(\sin\phi \cdot \cos\alpha + \sin\alpha . \cos\phi)$$

$$\therefore \boxed{P = \frac{W . \sin(\alpha+\phi)}{\cos\phi}}$$

This is the maximum Force required to keep the body in equilibrium when body is at the point of moving upwards.

Case II : When the Force applied horizontally

a) when the body is at the point of moving downward.



$$\boxed{P = W\tan(\phi - \alpha)}$$

This is the minimum force required to keep the body in equilibrium.

b) When the body is at the point of moving upwards.

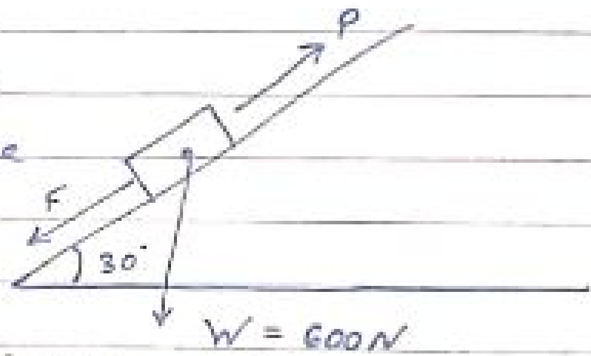
$$\boxed{P = W.\tan(\phi + \alpha)}$$

Maximum force to kelp the body in equilibrium

Problems

Q) A body weight 600N and is lying on a rough inclined plane which makes an angle of 30° with the horizontal. A force is applied to the body parallel to the plane. Find the minimum and maximum values of P to keep the body in equilibrium if the angle of friction is 25°.

Sol: Minimum value of force (P) for the equilibrium of the body.



$$= \frac{W\sin(\alpha \mp \phi)}{\cos\phi} = \frac{600\sin(30-25)}{\cos 25}$$

$= 57.7\ N$

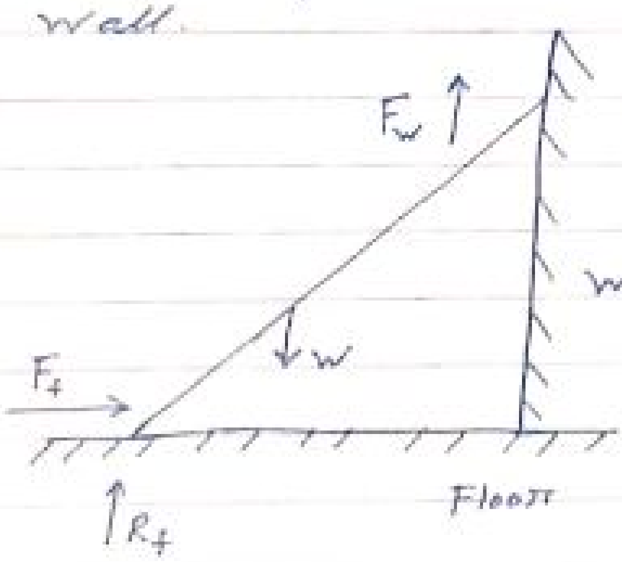
Maximum value of P for the equilibrium of body.

$$\frac{W\sin(\alpha+\phi)}{\cos\phi} = \frac{600\sin(30+25)}{\cos 25} = 542.3\ N$$

Ladder Friction

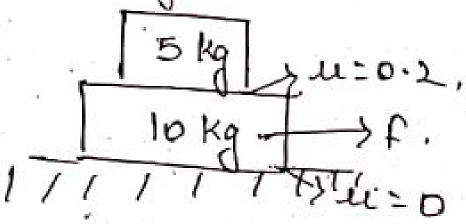
A ladder is a device or arrangment used for climbing the wall or roofs. It has two long uprights of wood or iron and connected by a number of cross bars. The cross bass is called rungs.

The upper end of the ladder tends to slip downwards for which the direction of force of friction between the ladder and the wall (F_w) acts upwards as shown. Again, the lower end of the ladder tends slip away from the wall, the direction of force of friction (F_f) between the floor and ladder will be towards the wall.



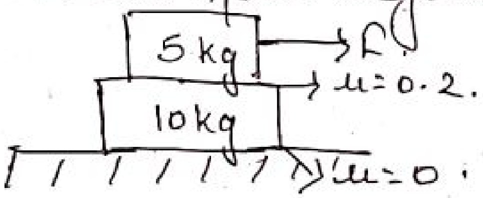
Q. Questions

Q. Find the maximum value of 'F' such that ~~the bl~~ both the blocks move together.



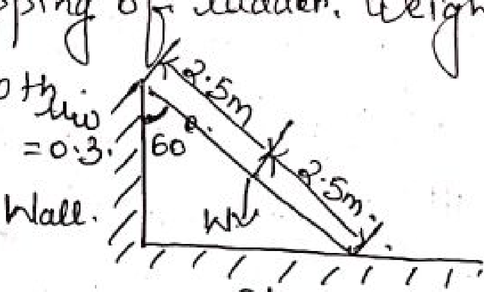
The coefficient of friction between both the blocks is "0.2", and in between the ground surface and the lower block, the coefficient of friction is "zero".

Q. Find the maximum force, 'F_{max}', such that both the blocks move together.



The coefficient of friction between both the blocks is 0.2, and the ground surface is frictionless.

Q. A ladder having a length of 5m is resting against a wall making an angle of 60° to it, having coefficient of friction as 0.3. Calculate the horizontal force required to be applied at the bottom end to avoid slipping of ladder. Weight of the ladder is 300N, & the floor is smooth.



Q.

Q. A ladder of weight 400 N and length 10m is supported on a smooth wall with it's lower end 4m away from the wall. The coefficient of friction between the ladder and the floor is 0.3. Find the frictional force at the floor.

Simple Machines

A simple machine is a mechanical device that changes the direction or magnitude of a force. It receives energy in some available form and uses it for doing a particular useful work.

Example: An IC engine converts the energy provided by petrol or diesel into mechanical or into motion of translation.

A machine does not work by itself. It does some useful work when energy is supplied to it.

Gear Drive

A gear may be defined as a wheel with projections or teeth cut on its rim or periphery.

- When belt or rope drives are used for power or motion transmission, slipping of belt occurs which reduces the velocity ratio of the system.

- In case of precision machines, a fixed or definite velocity ratio is required which can be achieved by the use of a positive drive like gear drive.

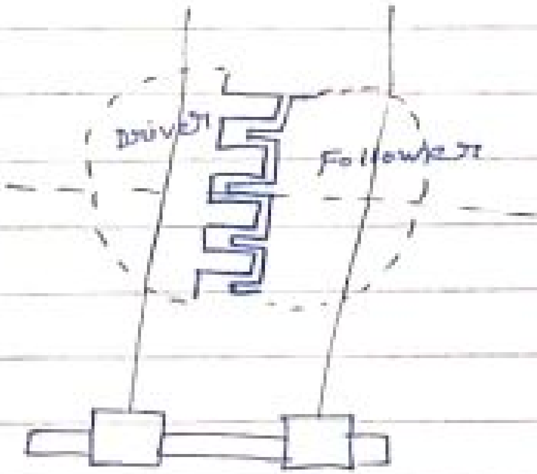
Velocity ratio of a simple Gear Drive

Velocity ratio is defined as the ratio of the velocities of the driver gear and the driven gear.

N_1 = speed of the driver gear

T_1 = Number of teeth on the driver gear

D_1 = Diameter of the pitch circle of the driver gear.



N_2, T_2, D_2 = Corresponding values for the driven gear.

P = Pitch of the driver & driven gear.

$\therefore$

$$\text{Velocity ratio} \Rightarrow \frac{N_2}{N_1} = \frac{D_1}{D_2} = \frac{T_1}{T_2}$$

Velocity ratio of simple gear train

$$\text{Velocity ratio} = \frac{\text{Speed of the driving Wheel}}{\text{speed of the driven wheel}}$$

$$= \frac{\text{Number of teeth on the driven wheel}}{\text{Number of teeth on the driving wheel.}}$$

Velocity ratio of Compound gear train

$$\text{Velocity ratio} \Rightarrow \frac{\text{Speed of the first driver gear}}{\text{Speed of the last driven gear}}$$

$$= \frac{\text{Product of number of teeth on the driven gears}}{\text{Product of the number of teeth on the driver gears}}$$

Simple Lifting machine

The machines which are used to lift heavy loads are called lifting machine. Such machines generally lift loads by applying a small effort in a convenient direction.

Mechanical advantage : It is the ratio of weight lifted to the effort applied.

$$MA = \frac{\text{Weight lifted}}{\text{Effort applied}} = \frac{W}{P}$$

Velocity ratio : It is the ratio of the distance moved by the effort to the distance moved by the load.

$$VR = \frac{\text{Distance moved by effort}}{\text{Distance moved by Load}} = \frac{y}{x}$$

~~Efficency~~ Efficiency of a machine:

It is the ratio of the useful work done by the machine to the work done on the machine.

$$\eta = \frac{\text{useful work done by machine}}{\text{Work done on the machine}}$$

$$= \frac{\text{Output of the machine}}{\text{input to the machine}}$$

$$= \frac{\text{mechanical Advantage (MA)}}{\text{Velocity ratio (VR)}}$$

Relation Between Efficency, Mechanical Advantage and Velocity Ratio of a Lifting Machine.

Let, W = Load lifted by the machine

P = effort to be applied

Y = Distance moved by effort, to lift the load

X = Distance moved by the load

$$\text{Mechanical Advantage (MA)} = \frac{\text{Load lifted}}{\text{Effort Applied}} = \frac{W}{P}$$

$$\text{Velocity Ratio (VR)} = \frac{\text{distance moved by effort}}{\text{distance moved by Load}}$$

$$= Y/x$$

Input to machine = effort applied × distance moved by the effort

$= Py$

Output of machine = Load lifed × distance moved by the load.

$= Wx$

~~Efficien~~ Efficiency $(\eta) = \frac{\text{output}}{\text{Input}} = \frac{Wx}{Py}$

$= \left(\frac{W}{P}\right) \times \frac{1}{(y/x)} = \frac{(W/P)}{(y/x)}$

$\Rightarrow \boxed{\eta = \frac{MA}{VR}}$

Problems

An effort of 25N is required to lift a load of 1kN. The distance moved by the effort is 8m, add the weight moves up through 100 mm. calculate mechanical advantage (MA), Velocity ratio (VR) and efficency of the machine.

$W = 1kN = 1000N$

$P = 25N$

$y = 8\ m \qquad x = 100mm = 0.1\ m$

$MA = \frac{W}{P} = \frac{1000}{25} = 40$

$VR = \frac{y}{x} = \frac{8}{0.1} = 80$

$$\eta = \frac{MA}{VR} = \frac{40}{80} = 0.5 = 50\%$$

2) In a weight lifting machine an effort of 50 N lifts a load 500 N. When the effort moves through 55 cm and the load moves up through 5 cm, Calculate MA, VR and ~~efficienc~~ efficiency of the machine.

Sol: $W = 500\,N$ $\quad P = 50\,N$ $\quad y = 55\,cm$ $\quad x = 5\,cm$

$$MA = \frac{W}{P} = \frac{500}{50} = 10$$

$$VR = \frac{y}{x} = \frac{55}{5} = 11$$

$$\eta = \frac{MA}{VR} = \frac{10}{11} = 0.9090 = 90.9\%$$

Example - 3 : *In a certain weight lifting machine, an effort of 40N is required to lift a load of 440 N. The distance moved by the effort is 65cm and the load is lifted through 4cm. Find out mechanical advantage, velocity ratio and efficiency of the lifting machine.*

Solution :

Given, $W = 440\,N$

$P = 50\,N$

$y = 65\,cm$

$x = 4\,cm.$

Mechanical advantage $MA = \frac{W}{P} = \frac{440}{40} = 11$

Velocity ratio $VR = \frac{65}{4} = 16.25$

Efficiency $\eta = \frac{MA}{VR} = \frac{11}{16.25} = 0.6769 = 67.69\%$

Example – 4 : *In a certain lifting machine, the velocity ratio is 15 and its efficiency is 70%. Find out the load to be lifted when an effort of 60 N is applied.*

Solution :

Given, VR = 15

η = 70% = 0.7

W = ?

P = 60 N.

We have,

$$\eta = \frac{MA}{VR} = \frac{W/P}{VR} = \frac{W}{P \times VR}$$

$$\Rightarrow \quad 0.7 = \frac{W}{60 \times 15}$$

$$\Rightarrow \quad W = 0.7 \times 60 \times 15 = 630 \text{ N}$$

9.5 : REVERSIBILITY OF A MACHINE AND CONDITION FOR REVERSIBILITY

Let (P) be the effort to be applied through a distance (y) to lift a load (W) through a distance (x). When the effort is removed, the machine does the work in the reverse direction. The machine is then called a reversible machine. Its action is called the reversibility of the machine. A pulley used to draw water from a well by bucket, may be considered as a reversible machine as the bucket falls back when the effort is removed.

If the work is not done by the machine in reverse direction, the machine is called irreversible machine or self-locking machine. Reversal of a machine is opposed by the frictional force acting on the moving parts of the machine. A screw jack is an irreversible machine. In an irreversible machine, some work is lost due to friction. The friction is given by :

Friction = Input – Output

= Py – Wx

When the effort is removed, in an irreversible machine, the friction is more than the output of the machine. We have,

$$Py - Wx > Wx$$

$$\Rightarrow \quad Py > 2\,Wx$$

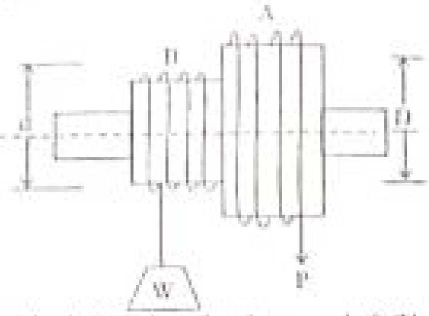
$$\Rightarrow \quad \frac{Wx}{Py} < \frac{1}{2} \Rightarrow \frac{w/p}{y/x} < \frac{1}{2} \quad \eta < \frac{1}{2} \text{ or } 50\,\%$$

Hence, the condition of irreversibility of a machine is that the efficiency of a machine should be less then 50%. Similarly the condition for reversibility of a machine is that the efficiency of the machine should be more than 50%.

Simple Wheel and AXLE

SIMPLE WHEEL AND AXLE

Refer the figure shown.



In the figure, wheel A and axle B are keyed to the same shaft. The entire assembly is mounted on ball bearings in order to reduce the frictional resistance. A string is wound round the axle B, which carries the load to be lifted. Another string is wound round the wheel A in the opposite direction to that of the string on B, and the effort P is applied to it.

Let, D = diameter of the wheel A.

d = diameter of the axle B

W = load to be lifted.

P = effort to be applied.

As the wheel A and the axle B are keyed to the same shaft, when the wheel makes one revolution, the axle will also make one revolution.

Again, as the strings are wound in opposite directions, the downward movement of the effort (P) will make the load (W) move up.

In one revolution of the wheel (A), the effort moves through (πD) and at the same time, the axle B also makes one revolution and the load moves through (πd).

$$\therefore \quad \text{Velocity ratio (VR)} = \frac{\text{distance moved by effort}}{\text{distance moved by load}}$$

$$= \frac{\pi D}{\pi d} = \frac{D}{d}$$

$$\text{Mechanical advantage (MA)} = \frac{\text{load lifted}}{\text{effort applied}} = \frac{W}{P}$$

$$\text{Efficiency } (\eta) = \frac{MA}{MR}$$

Q) In a simple wheel and axle arrangement, a load of 30N is lifted an effort of 10 N. The efficency is 90%. If the diameter of the wheel is 280 mm, Find the diameter of the axle.

ns: $W = 30N$ $\quad D = 380\text{ mm}$ $\quad d = ?$

$P = 10N$

$\eta = 0.9$

$$\text{Mechanical advantage (MA)} = \frac{W}{P} = \frac{30}{10} = 3$$

Velocity ratio (VR) $= \dfrac{MA}{\eta}$

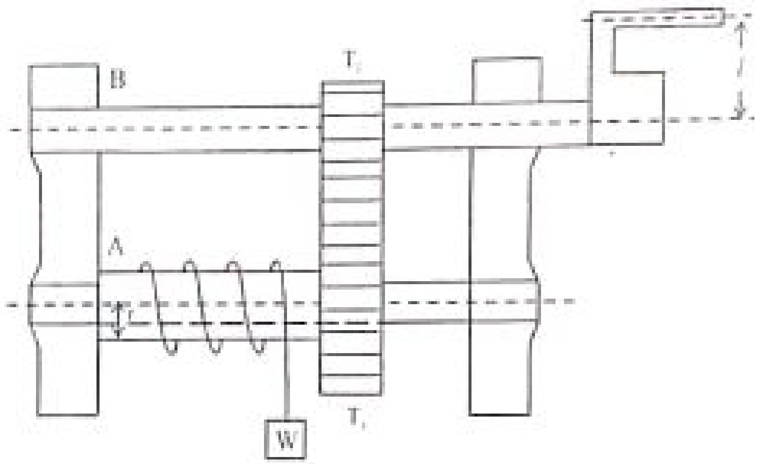
So, $VR = \dfrac{3}{0.9} = 3.334$

But, $VR = \dfrac{D}{d}$

$\Rightarrow 3.334 = \dfrac{380}{d}$

$\Rightarrow d = 113.97$ mm

9.11 : SINGLE PURCHASE CRAB WINCH



In single purchase crab winch, a rope is fixed to the drum and is wound a few turns round it. The free end of the rope carries the load W. A toothed wheel A is rigidly mounted on the load drum. Another toothed wheel B, caned pinion, is geared with the toothed wheel A as shown in the above figure.

The effort is applied at the end of the handle to rotate it.

Let T_1 = No. of teeth on the main gear A.

T_2 = No. of teeth on the pinion B.

l = length of the handle.

r = radius of the load drum.

W = Load lifted

P = effort applied to lift the load.

Now, distance moved by the effort in one revolution of the handle, $= 2\pi l$ .....(1)

No. of revolutions made by the pinion B = 1

And no. of revolutions made by the wheel $A = \dfrac{T_2}{T_1}$

Hence, No. of revolutions made by the load drum $= \dfrac{T_2}{T_1}$

And distance moved by the load $= 2\pi r \times \dfrac{T_2}{T_1}$ ....(2)

$\therefore$ V.R. $= \dfrac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$

$= \dfrac{2\pi l}{2\pi r \times \dfrac{T_2}{T_1}} = \dfrac{l}{r} \times \dfrac{T_1}{T_2}$ ....(3)

Now, M. A. $= \frac{W}{P}$ and efficiency $\eta = \frac{MA}{VR}$

Example :

Following are the details about a single purchase crab winch

Length of lever = 700 mm

Number of pinion teeth = 12

Number of spur gear teeth = 96

Diameter of load axle = 200 mm

It is observed that an effort of 60N lifts a load of 1800 N and an effort of 120N lifts a load of 3960 N. Find the law of machine. Find the efficiency of the machine in both the cases.

Solution :

(a) **Law of Machine**

l = length of lever = 700 mm

T_2 = No. of pinion teeth = 12

T_1 = No. of spur gear teeth = 96

Diameter of load axle = 200 mm

Radius of load axle = 100 mm

When $P_1 = 60$ N, $W_1 = 1800$ N

When $P_2 = 120$ N, $W_2 = 3960$ N

Substituting the value of P_1, P_2, W_1 and W_2 in the law of machine, we get :

$60 = m \times 1800 + C$ .... (1)

$120 = m \times 3960 + C$ .... (2)

Subtracting equation (i) from equation (2), we get :

$60 = m \times 2160$

$$\Rightarrow m = \frac{60}{2160} = \frac{1}{36}$$

Now substituting this value of m in equation (1), we get :

$$60 = \left(\frac{1}{36} \times 1800\right) + C = 50 + C$$

$$\Rightarrow C = 60 - 50 = 10$$

Now substituting the value of $m = \frac{1}{36}$ and C = 10 in the law of machine,

we get :

$$P = \frac{1}{36} W + 10 \quad \text{(Ans.)}$$

(b) **Efficiencies of the machine in both the cases :**

Velocity ratio V.R. $= \frac{l}{r} \times \frac{T_1}{T_2} = \frac{700}{100} \times \frac{96}{12} = 56$

Mechanical advantage in the first case,

$$MA = \frac{W_1}{P_1} = \frac{1800}{60} = 30$$

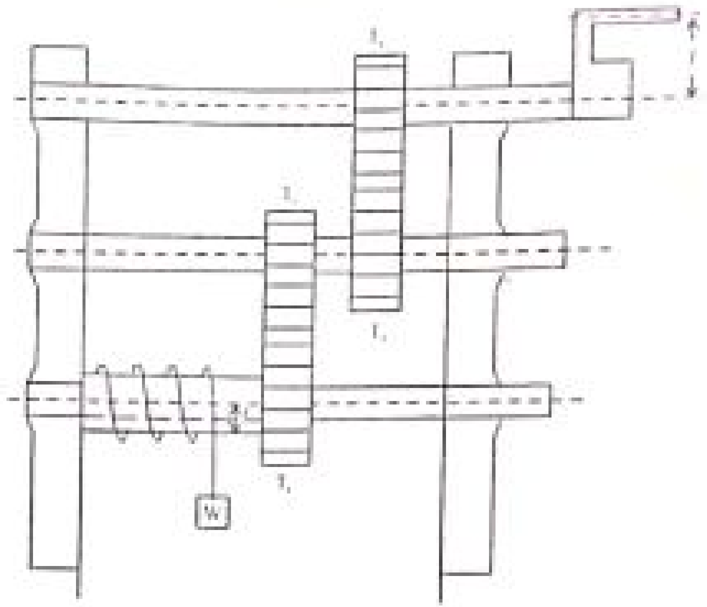
$\therefore$ efficiency $\eta_1 = \frac{MA}{VR} = \frac{30}{56} = 0.536 = 53.6\%$ (Ans.)

Similarly, mechanical advantage in the second case,

$$MA = \frac{W_2}{P_2} = \frac{3960}{120} = 33$$

$\therefore$ efficiency $\eta_2 = \frac{MA}{VR} = \frac{33}{56} = 0.589 = 58.9\%$ (Ans.)

DOUBLE PURCHASE CRAB WINCH



This is an improved form of single purchase crab winch, in which the velocity ratio is intensified with the help of one more spur wheel and a pinion. In this case, there are two spur wheels of teeth T_1 and T_3 along with two pinion T_2 and T_4.

The arrangement of spur wheels and pinions are such that the spur wheel with T_1 gears with the pinion of teeth T_2. Similarly, the spur wheel with T_3, gears with the pinion of the teeth T_4. The effort is applied to a handle as shown in the figure above.

Let, T_1 and T_3 = No. of teeth on spur wheels.

T_2 and T_4 = No. of teeth of the pinions.

l = length of the handle.

r = radius of the load drum.

W = Load lifted.

P = Effort applied to lift the load, at the end of the handle.

Now, distance moved by the effort in one revolution of the handle, = $2\pi l$ .....(1)

No. of revolutions made by the pinion 4 = 1.

and No. of revolutions made by the wheel 3 $= \dfrac{T_4}{T_3}$

No. of revolutions made by the pinion 2 $= \dfrac{T_4}{T_3}$

and No. of revolutions made by the wheel 1 $= \dfrac{T_2}{T_1} \times \dfrac{T_4}{T_3}$

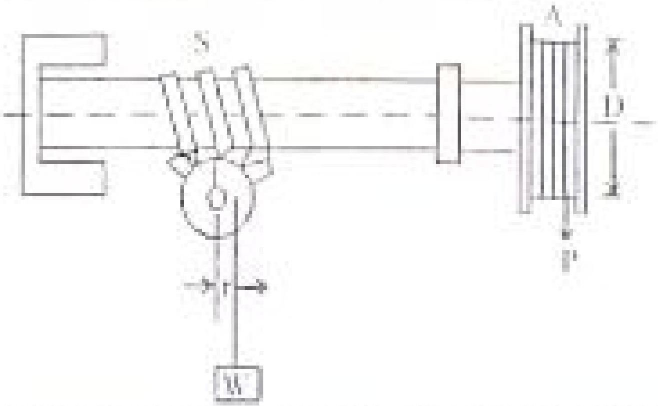
Hence, distance moved by the load $= 2\pi r \times \dfrac{T_2}{T_1} \times \dfrac{T_4}{T_3}$ ....(2)

$$\therefore \quad VR = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

$$= \frac{2\pi l}{2\pi r \times \dfrac{T_2}{T_1} \times \dfrac{T_4}{T_3}} = \frac{l}{r}\left(\frac{T_1}{T_2} \times \frac{T_3}{T_4}\right)$$

Now, M.A. $= \dfrac{W}{P}$ and efficiency $\eta = \dfrac{MA}{VR}$

Worm and Worm Wheel



It consists of a square threaded screw. The worm S and a toothed wheel called worm wheel are geared with each other as shown. Wheel A is attached to the worm over which a rope passes, as shown in the fifure. A load drum is securely mounted on the worm wheel.

Let, D = Diameter of the effort wheel.

r = radius of the load drum

W = Load lifted.

P = effort applied to lift the load.

and T = No. of teeth on the worm wheel.

The distance moved by the effort in one revolution of the wheel or handle sometimes a handle is also attached to the worm instead of the wheel) = πD. ......(1)

If the worm is single-threaded i.e., for one revolution of the wheel A, the screw S pushes the worm wheel through one teeth, then the load drum will move through $\frac{1}{T}$ revolutions.

The distance through which the load will move = $\frac{2\pi r}{T}$ ....(2)

$$\text{Now, } VR = \frac{\text{Distance moved by the effort}}{\text{Distance moved by the load}}$$

$$= \frac{\pi D}{\frac{2\pi r}{T}} = \frac{DT}{2r} \quad ....(3)$$

$$\text{and } M.A = \frac{W}{P}$$

$$\text{Now, efficiency } \eta = \frac{MA}{VR}$$

If the worm is doubled-threaded i.e., for one revolution of wheel A, the screw S pushes the worm wheel through two teeth, then

$$VR = \frac{DT}{2 \times 2r} = \frac{DT}{4r}$$

If the worm is n threaded, then $VR = \frac{DT}{2nr}$

Numerical of Worm and Worm wheel.

Example :

In a double threaded worm and worm wheel, the number of teeth on the worm wheel is 60. The diameter of the effort wheel is 250 mm and that of the load drum is 100 mm. Calculate the velocity ratio. If the efficiency of the machine is 50%, determine the effort required to lift a load of 100 N.

Solution :

Data given :

No. of threads n = 2

No. of teeth on the worm wheel T = 60

Diameter of effort wheel = 250 mm.

Diameter of load drum = 100 mm.

radius of load drum = 50 mm.

Efficiency η = 50% = 0.5

and load to be lifted W = 300 N.

Velocity ratio of the machine,

$$VR = \frac{DT}{2nr} = \frac{250 \times 60}{2 \times 2 \times 50} = 75 \quad \text{(Ans.)}$$

Now, $MA = \frac{W}{P} = \frac{300}{P}$ and efficiency $\eta = \frac{MA}{VR}$

$$\Rightarrow 0.5 = \frac{\frac{300}{P}}{75}$$

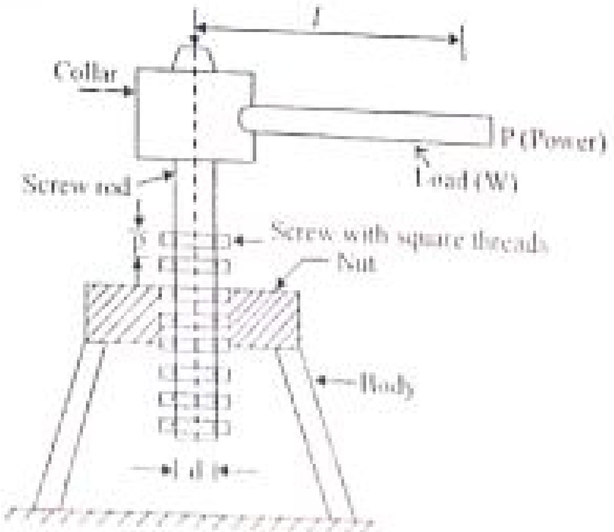
$$\Rightarrow P = \frac{4}{0.5} = 8\,\text{N} \quad \text{(Ans)}$$

Screw Jack

: SCREW JACK

A Simple Screw Jack consists of a Screw fitted in a nut that forms the body of the jack. It a simple machine that lifts heavy loads through short distances by the application of little effor applied at its handle. It works on the principle similar to that of an inclined plane.

Figure shows a simple Screw Jack.



Let, L = Length of the effort arm

p = pitch of the screw

W = load lifted

P = effort applied at the end of the lever

In one rotation of the handle the effort moves through a distance of $2\pi L$.

Now, distance moved by the load = pitch of thread = p

$$VR = \frac{\text{distance moved by the effort}}{\text{distance moved by the load}}$$

$$\Rightarrow \quad VR = \frac{2\pi L}{P}$$

$$MA = \frac{W}{P}$$

Efficiency, $\eta = \dfrac{MA}{VR}$

Solved Examples

Example – 1 : ***A Screw Jack has a thread of pitch equal to 10mm. Find the effort to be applied at the end of the handle 500mm long to lift a load of 1800 N. The efficiency of the system is 52%.***

Solution :

Given
p = 10 mm
L = 500 mm
W = 1800 N
η = 0.52
P = ?

$$VR = \frac{2\pi L}{P} = \frac{2\pi \times 500}{10} = 314.16$$

$$MA = \frac{W}{P} = \frac{1800}{P}$$

$$\eta = \frac{MA}{VR} \Rightarrow 0.52 = \frac{1800}{P \times 314.16} \Rightarrow P = 11.018\text{ N.}$$

Example – 2 : ***In a simple Screw Jack the pitch of the thread is 10 mm. Calculate the effort required to lift a load of 1500 N, when applied at the end of a handle 600 mm long. The efficiency of the system is 60 percent.***

Solution :

Given
p = 10 mm
effort P = ?
W = 1500 N
L = 600 N
η = 0.6

$$VR = \frac{2\pi L}{p} = \frac{2\pi \times 600}{10} = 376.99$$

$$MA = \frac{W}{P} = \frac{1500}{P} \quad \eta = \frac{MA}{VR} = \frac{W}{P \times VR}$$

$$\Rightarrow \quad 0.6 = \frac{1500}{P \times 376.99}$$

Law of Machine

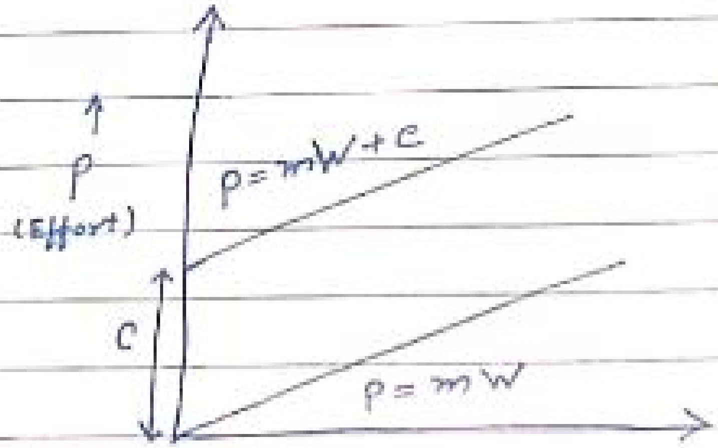
The law of machine outlines the relationship between the effort applied and the load lifted. Hence, the law of machine may be defined as the relationship between the effort applied and load lifted.

Various efforts are applied to lift different loads and a graph is plotted between effort and load.

For a machine with effort constant velocity ratio is given by:

$$\boxed{P = mW + C}$$

m = slope of the straight line
C = intercept of the line on y-axis.



* In a ideal machine, there is no friction.

So, $C = 0$.

$\therefore \quad P = mW + 0$

$\quad\quad = mW$

Dynamics

Kinematics : It is that branch of dynamics which establishes a relationship between displacement, Velocity, accleration and time of a given motion without considering the forces causing motion.

Kinetics: It is the branch of dynamics which establishes a relationship between the mass and motion of the body and the forces causing the motion. It predicts the type of motion by a given force system. It also determines the force system required for a prescribed motion.

Newton's law of Motion

First law of Motion

" Everybody continues in its State of rest or of uniform motion in a straight line, unless compelled Some external force to change that State."

- Every body continues in its State of rest unless Compelled by Some external force to change the state of rest.
- Everybody continues in it state of uniform motion in a Straight line unless compelled by Some external force to change that state.

A ball rolling on the floor comes to rest after sometime due to air resistance and the friction between the floor and the ball. If the air resistance and the force of friction are eliminated, the ball will continue to move in straight line.

This second part is the definition of inertia and hence, the first law of motion can also be termed as law of inertia.

A body at rest, tends to remain at rest. It is the inertia of rest.

A body in uniform motion in a straight line, tends to retain motion. It is the inertia of motion.

(b) Second low of motion :

"The rate of change of momentum is directly proportional to the impressed force and takes place in the direction in which the impressed force acts.

This law relates the rate of change of momentum and the external force or impressed force.

Let, m = mass of the body.

u = initial velocity of the body

v = Final velocity of the body

a =acceleration

t = time in seconds in which the velocity changes from u to v.

F= force that changes the velocity from u to v in t seconds.

Now, for the body moving in a straight line,

initial momentum = mu

Final momentum = mv

$\therefore$ Rate of change of momentum $= \frac{mv - mu}{t} = \frac{m(v-u)}{t} = ma$

Where, $\frac{v-u}{t} = a$.

According to Newton's second law of motion;

rate of change of momentum α applied force.

$\Rightarrow$ F α ma

$\Rightarrow$ F = K.ma ...(1)

Where, K = a constant of proportionality.

Now, if a unit force is so choosen to act on a unit mass to produce unit acceleration in it, we have,

F = K.ma

$\Rightarrow$ 1 = K × 1 × 1

$\Rightarrow$ K = 1

Putting this in equation (1), we get

F = ma.

The unit of force obtained from the product of the mass and acceleration is known as absolute unit as this magnitude of force remains the same anywhere in the universe. In SI system of units, the unit of force is Newton, briefly written as N.

A Newton is the force which when acts on a mass of 1 kg., produces an acceleration of 1 m/sec² in the direction in which it acts.

Newton's second law of motion is also known as law of dynamics. It has two parts which are as follows :

(i) Acceleration is induced in a body only when some force is applied on it.

(ii) The applied force is proportional to the product of mass of the body and the acceleration induced in it.

(c) Third law of motion :

"To every action, there is always an equal and opposite reaction".

If a body exerts a force P on another body, the second body will exert the same force P on the first body in the opposite direction. The force exerted by the first body on the second body is called action where as the force exerted by the second body on the first body is called reaction.

11.3. D'ALEMBERT'S PRINCIPLE

"If a rigid body is acted upon by a system of forces, the system of forces is in dynamic equilibrium with the inertia force of the body".

Let, P = resultant of a number of forces acting on the rigid body of mass m.

Then, this resultant (P) will move the body with an acceleration (a) in its own direction. We have,

$$P = ma \qquad ...(1)$$

The body will be at rest if a force equal to (ma) is applied in reverse direction. Hence, for dynamic equilibrium of the body, the sum of the resultant force and the reversed force will be equal to zero. We have

$$P - ma = 0 \qquad ...(2)$$

The force (-ma) is known as inertia force or reversed effective force.

Equation (1) is the equation of dynamic and equation (2) is the equation of statics. The equation (2) is known as the equation of dynamic equilibrium under the action of P. This principle is known as D'Alembert's principle.

Solved Examples

Example – 1 : *Determine the acceleration produced in a body of 70 kg mass when it is subjected to a force of 150N. Use D'Alembert's principle.*

Solution :

Given, $P = 150$ N

$M = 70$ kg

As per D'Alembert's principle, the inertia force (ma) is applied in the reverse direction

Now, $P - ma = 0$

$\Rightarrow 150 - 50 \times a = 0$

$\Rightarrow a = 3\text{m/sec}^2$

11.5 : WORK

When a force acts on a body, the body moves through certain distance.

The amount of work done depends upon the magnitude of the force and the amount of displacement of the body.

The amount of work done is equal to the product of force and the displacement in the direction of the force.

Let, P = force acting on the body

and s = distance through which the body moves

Now, the work done, $W = P \times s$

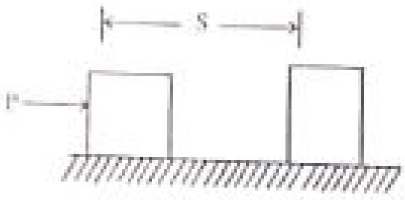


Fig.(a)

Force and displacement are collinear

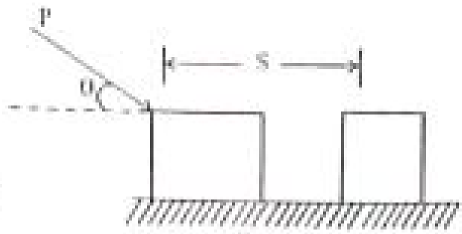


Fig.(b)

Force and displacement are not collinear

Sometimes the force and the displacement are not collinear.

In such a case, work done is expressed as the product of the component of the force in the direction of motion and the displacement. Refer Fi (b).

Here, work done $W = P \cos \theta \times s$

If $\theta = 90°$, $\cos \theta = 0$ and hence, there will be no work done. We may conclude that if force and displacement are at right angles to each other, work done will be zero.

When the point of application of the force moves in the direction of motion of the body, work is said to be done by the force. Work done by the force is taken as +ve.

Similarly, when the point of application of the force moves in a direction opposite to that of the motion of body, work is said to be done against the force. Work done against the force is taken as –ve.

As work is the product of force and displacement, the units of work depend upon the units of force and displacement. Work is expressed in N-m or KN–m.

One newton-meter or 1N-m is the work done by a force of 1N in moving the body through 1m. 1N-m.= 1 Joule. or 1J.

Similarly, one kilo Newton–meter or 1KN-m is the work done by a force of 1KN in moving abody through 1m.

1KN – m = 1 kilo Joule or 1 KJ

11.6 : FORCE – DISPLACEMENT DIAGRAM

This is the graphical representation of work done. Work done by a force can be represented by the area under the force-displacement curve or diagram.

Now refer figure (a) (b) (c) (d) and (e).

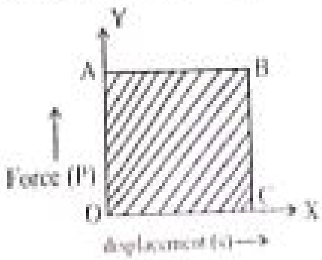


Fig. (a)

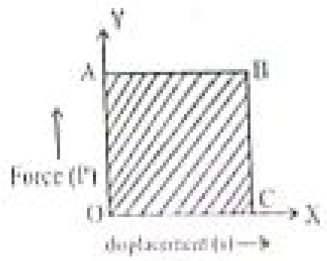


Fig. (b)

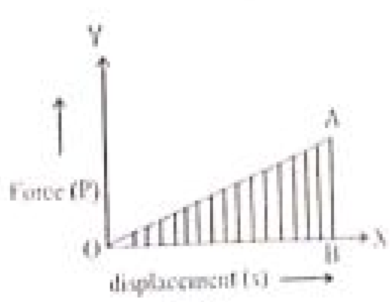


Fig.(c)

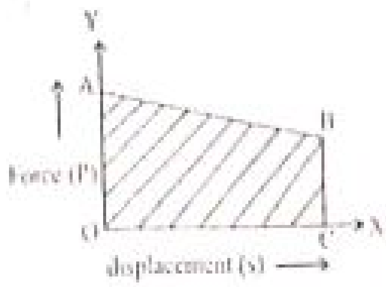


Fig.(d)

Power

Power is defined as the rate of doing work. Power is calculated by dividing the work done by time.

The unit of power is Watt (W), kilowatt (kW) in SI unit.

$1 \text{ Watt} = 1 \frac{\text{Joule}}{\text{second}}$ or $1 W = 1 \frac{J}{S}$ or $1 \frac{N\text{-}m}{S}$

$1 kW = 10^3 \text{ Watt}$, $1 MW = 10^6 \text{ Watt}$.

Power is the measure of performance of engine. In case engines, the following two terms are used for powers.

Indicated power (IP): It is the actual power generated in the engine cylinder.

Indicated power is the actual power generated in the engine cylinder and it is also considered as the power that is supplied to the engine in terms of calorific value of fuel.

(b) **Brake Power (B.P.)**

The total power developed by the engine cylinder is not available for work. This is due to the fact that a part of the power developed by the engine cylinder is utilized in overcoming friction of the moving parts of the engine. Hence, the power available for useful work or the total output of the engine is (IP – losses). This is called brake power

11.9 : EFFICIENCY OF AN ENGINE

The efficiency or mechanical efficiency is defined as the ratio of brake power to indicated power.

We have, $\eta = \frac{BP}{IP}$

11.10 : ENERGY

Energy may be defined as the capacity for doing work. Since energy of a machine is measured by the work it can do, therefore the unit of energy is the same as that of work.

In SI system, energy is expressed in joules or KiloJoule. There are two types of mechanical energy –

(a) **Potential Energy (P.E.)**

It is the energy possessed by a body by virtue of its position.

A body, at some height above the ground level, possesses potential energy as it can do work by falling on the ground.

If a body of mass (m) is raised to a height (h) above the ground level, the work done in raising the body is :

= weight of the body × distance through which it moved

= (mg) × h = mgh.

This work is stored in the body as potential energy. While coming down, the body can a do work equal to (mgh). Potential energy is zero when the body is on the earth.

(b) **Kinetic Energy (KE)**

It is the energy possessed by a body by virtue of its motion.

We can measure the kinetic energy of a body form the work done by the body against the external force to stop it.

Let, m = Mass of the body.

u = Velocity of the body at any instant.

P = External force applied.

a = Retradation of the body.

s = distance traveled by the body before coming to rest.

As the body comes to rest, its final velocity v = 0

We have, P = ma

But, $v^2 - u^2 = -2as$ (For retardation)

$0 - u^2 = 2.(-a)s \Rightarrow u^2 = 2as$

$u^2 = 2 \times \frac{P}{m} \times s$ $\left[As.\ a = \frac{P}{m}\right]$

But Work done W = Force × distance

= P×S

nce W = P×S or $KE = P \times S = \frac{1}{2} m u^2$

When initial velocity is (V) ~~and~~ instead of (u)

∴ $\boxed{KE = \frac{1}{2} m v^2}$

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11.17 : FORCE, MOMENTUM AND IMPULSE

(a) **Force**

Force is an external agent that tends to change the state of rest or of uniform motion of a body. It accelerations or retards the motion of a body. It is a vector quantity. A force is specified by its magnitude, line of action, point of application and direction.

Newton's second law of motion provides the relationship between motion and force.

Force α rate of change of momentum.

(b) **Momentum**

If is the product of mass and velocity of a body. It represents the energy of motion stored in a moving body.

If, m = mass of a moving body in kg.

and v = velocity of the body in m/sec.,

the momentum of the body = mv $kg - m/_{sec}$

(c) **Impulse**

According to the second law of motion,

F = ma

$$\Rightarrow \quad F = m.\frac{dv}{dt} = \frac{d}{dt}(mv)$$

$$= \frac{mv - mu}{t} \qquad ...(1)$$

Where, v = final velocity

and u = initial velocity.

The product of mass and velocity is the momentum of the body.

We have, mv = momentum of the body after time (t)

mu = momentum of the body at the beginning of motion.

From equation (1), we see that the change in linear momentum per unit time is directly proportional to the external force or applied force and takes place in the direction of the force.

Again, from equation (1), we have,

$$F \times t = mv - mu \qquad ...(2)$$

$$= m(v - u)$$

The product of force and the time during which it acts, is called impulse of force.

Hence, impulse = F × t = mv – mu, i.e. impulse is equal to change in momentum.

The equation (2) represents impulse – momentum relation.

11.18 : LAW OF CONSERVATION OF LINEAR MOMENTUM

It states "total momentum of any system (group of objects) always remains constant if no external force acts on it".

Momentum along a straight line is called linear momentum.

$m_1 \xrightarrow{u_1}$ $m_2 \xrightarrow{u_2}$ m_1 m_2 $m_1 \xrightarrow{v_1}$ $m_2 \xrightarrow{v_2}$

It may also be stated as. "for an isolated system, total momentum of the system is constant".

Suppose a body a of mass (m_1) moving with velocity (u_1) collides with another body B of mass (m_2) moving with velocity (u_2). Let (v_1) and (v_2) be the velocities of body A and B respectively, after the collision. We have :

Total momentum before collision = $m_1u_1 + m_2u_2$

and total momentum after collision = $m_1v_1 + m_2v_2$

Now, according to the law of conservation of linear momentum

momentum before collision = momentum after collision.

$\Rightarrow \quad m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

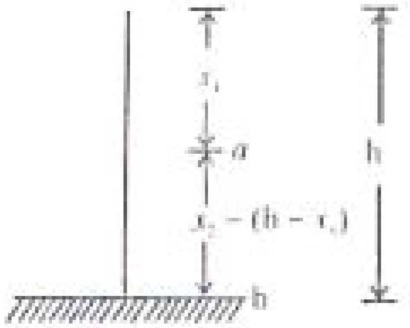
11.19. LAW OF CONSERVATION OF ENERGY

It states, "energy cannot be created, nor can it be destroyed, but it can be transformed from one form to the other."

It may also be stated as, "the total energy possessed by an object remains constant provided no energy is added to or substracted from it".

Suppose a body of mass (m) is at a height (h) above the ground level. At this position, the kinetic energy (KE) of the body,

$$= \frac{1}{2}mv^2 = 0 \text{ (as, v = 0)}$$



But the potential energy (PE) of the body = mgh

Total energy of the body = KE + PE = 0 + mgh = mgh

Now, let the body falls through a distance of (x_1) to point 'a'. We have

$v^2 - u^2 = 2as$

$v_1^2 - (0)^2 = 2gx_1$

[v_1 = velocity of fall of the body and u = initial velocity = 0]

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$\Rightarrow \quad v_1^2 = 2gx_1$

KE of the body at point 'a' $= \frac{1}{2}mv_1^2 = \frac{1}{2}m \times 2gx_1 = mgx_1$

and PE of the body at point 'a' = mg (h − x_1) mgh − mgx$_1$

Total energy of the body at point 'a'

= KE + PE

= mgx$_1$ + (mgh − mgx$_1$) = mgh

= Total energy of the body at the height (h).

Again, let the body falls on the ground level to point 'b'.

Let v^2 = velocity of fall

We have $v^2 - u^2 = 2as$

$v_2^2 - (0)^2 = 2gh$

KE of the body at point 'b' $= \frac{1}{2}mv_2^2 = \frac{1}{2}m \times 2gh = mgh$

and PE of the body at point 'b' = mg × 0 = 0

Total energy of the body = KE + PE = mgh + 0 = mgh

= Total energy of the body at height (h)

= Total energy of the body at point 'a'

It shows the sum of KE and PE of a body remains constant under the action of gravity. Even if KE and PE change individually, their sum remains constant.

11.20 : COLLISION OF ELASTIC BODIES

The term collision means the interaction or the contact between two bodies for a short period of time. The bodies produce impulsive forces on each other during collision. It is seen that this impulsive force is larger than any other finite force present or produced during the process. The act of collision between two bodies that takes place in a short period of time and during which the bodies exert very large forces on each other, is known as impact. The bodies come to rest for a moment immediately after collision. During the phenomenon of collision, the bodies tend to compress each other. The bodies tend to regain their actual shape and size after impact, due to elasticity or elastic property. The process of getting back the original shape is called restitution.

The time of compression is the time taken by the two bodies in compression, immediately after collision. The time of restitution is the time of regaining the original shape after collision. The period of impact or the period of collision is the sum of the time of compression and restitution.

11.21 : NEWTON'S LAW OF COLLISION OF ELASTIC BODIES AND COEFFICIENT OF RESTITUTION

Newton's law of collision of elastic bodies states that when two moving bodies collide with each other, thier velocity of separation bears a constant ratio to their velocity of approach

Let us consider two bodies A and B of masses (m_1) and (m_2) respectively move along the same line and produce direct impact. Direct impact is the impact in which two bodies move along the same line of impact before collision. The line of impact is the line joining the centers of two bodies in collision passing through the point of contact of the bodies.

Let u_1 = initial velocity of the body A.

u_2 = initial velocity of the body B.

v_1 = final velocity of the body A after collision.

v_2 = final velocity of the body B after collision.

impact will take place when $u_1 > u_2$.

Hence, the velocity approach = $u_1 - u_2$.

The two bodies separate from each other, after impact if $v_2 > v_1$.

Hence, the velocity of separation = $v_2 - v_1$.

According to the Newton's Law of collision of elastic bodies,

$(v_2 - v_1) \alpha (u_1 - u_2)$

$\Rightarrow \quad (v_2 - v_1) = e(u_1 - u_2)$

Where, e = a constant of proportionality known as coefficient of restitution.

The value of (e) lies between 0 and 1. If e = 0, it indicates that the two bodies are inelastic. If e = 1, it indicates that the two bodies are perfectly elastic.

11.22 : DIRECT COLLISION OF TWO BODIES

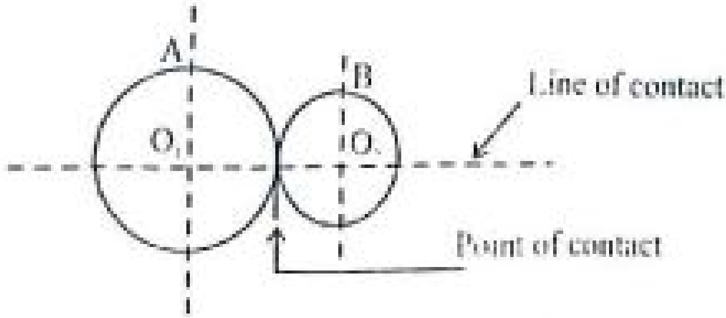
Consider two bodies A and B having a direct impact as shown in the figure.

Let, m_1 = Mass of the body A.

u_1 = initial velocity of the body A.

v_1 = Final velocity of the body A.

m_2, u_2, v_2 = Corresponding values for the body B.



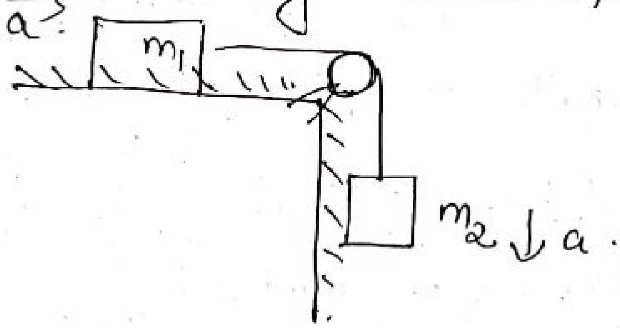
According to the law of conservation of linear momentum, we have,

$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$

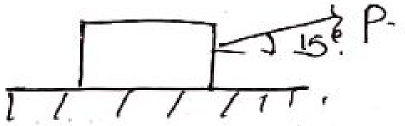
If one body is at rest initially, then such a collision is also called direct impact.

Questions.

Q. Two blocks of masses m_1 and m_2 are connected by a flexible by inextensible string as shown in the figure. Assume the coefficient of friction between block m_1 and the horizontal surface to be μ, find the acceleration of the masses and the tension in the string. Assume $m_1 = 10$ kg, $m_2 = 5$ kg & $\mu = 0.25$



Q. Find 'p' required to accelerate the block shown in figure below at $2.5 m/s^2$. Take $\mu = 0.3$.



Q. A constant force of 50N is applied to a 20kg block for 10 seconds. (a) What is the impulse acting on the block? (b) What is the change in momentum of the block? (c) What is the final speed of the block, if it was initially at rest? (d) What is the final speed of the block if it was originally moving at 15m/s?

Q. A 0.20 kg ball was strucked by a baseball bat from rest upto a speed of 35 m/s. The ball was in contact with the bat for 0.02 seconds. (a) What is the change in momentum of the ball? (b) What was the impulse exerted on the ball (c) Calculate the average force.

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exerted on the ball by the bat ..

Q. A 0.2kg tennis moves east at a speed of 50m/s and strikes a wall. The ball bounces back at a speed of 50m/s. The contact time between the wall and the ball was 0.015 seconds. What average force was exerted by the wall on the ball?

Q. A 70 N force is applied horizontally to a 10kg block at rest for a displacement of 200m across a frictionless surface. (a) How much work is done by the ~~wo~~ force? (b) What is the final kinetic energy? (c) How fast is the block moving? (d) What is the acceleration of the block in the horizontal direction? (e) Use kinematics to calculate the final speed of the block.

Q (a) How much work is required to accelerate a 1500kg car from 15m/s to 40m/s? (b) What is the average net force acting on the car if it reaches a final speed of 40m/s while travelling a distance of 275m?

Centre of gravity

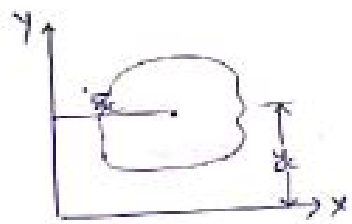
Centre of gravity: It is that point through which the resultant of the distributed gravity force passes regardless of the orientation of the body in space.

- As the point through which resultant of force of gravity (weight) of the body acts.

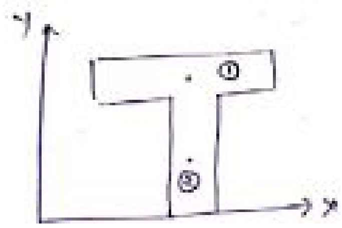
Centroid: Centrroid of an area lies on the axis of symmetry if it exits.

Centre of gravity is applied to bodies with mass and weight and centroid is applied to plane areas.

$$x_c = \sum A_i x_i$$
$$y_c = \sum A_i y_i$$

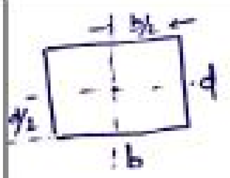
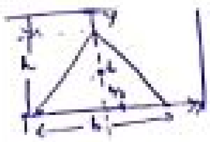
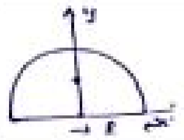
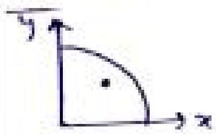


$$x_c = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$$
$$y_c = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$$

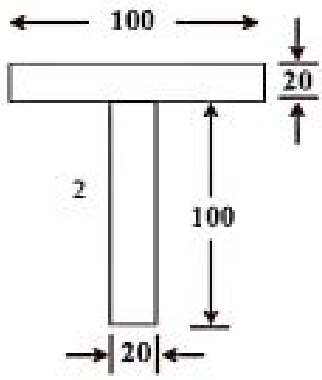


$$x_c = y_c = \frac{\text{Moment of area}}{\text{Total area}}$$
$$x_c = \frac{\int x.dA}{A}$$
$$y_c = \frac{\int y.dA}{A}$$

Centroids of different figures

Shape	Figure	$\bar{x}$	$\bar{y}$	Area
Rectangle		$\frac{b}{2}$	$\frac{d}{2}$	bd
Triangle		0	$\frac{h}{3}$	$\frac{bh}{2}$
Semicircle		0	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{2}$
Quarter circle		$\frac{4R}{3\pi}$	$\frac{4R}{3\pi}$	$\frac{\pi r^2}{4}$

Problem 3: Find the centroid of the T-section as shown in figure from the bottom.

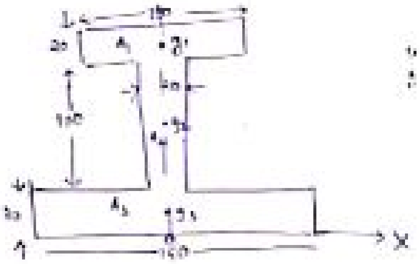


Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
2000	0	110	10,000	22,0000
2000	0	50	10,000	10,0000
4000			20,000	32,0000

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2} = \frac{32,0000}{4000} = 80$$

Due to symmetry, the centroid lies on Y-axis and it is at distance of 80 mm from the bottom.

Problem 4: Locate the centroid of the I-section.



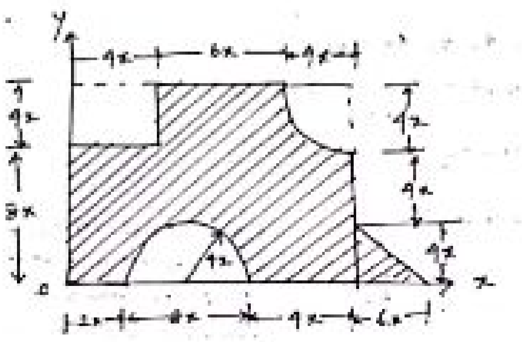
As the figure is symmetric, centroid lies on y-axis. Therefore, $\bar{x} = 0$

Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
2000	0	140	0	280000
2000	0	80	0	160000
4500	0	15	0	67500

$$y_c = \frac{\sum A_i y_i}{A_i} = \frac{A_1 y_1 + A_2 y_2 + A_3 y_3}{A_1 + A_2 + A_3} = 59.71mm$$

Thus, the centroid is on the symmetric axis at a distance 59.71 mm from the bottom.

Problem 5: Determine the centroid of the composite figure about x-y coordinate. Take x = 40 mm.



A_1 = Area of rectangle = 12x.14x=168x^2
A_2 = Area of rectangle to be subtracted = 4x.4x = 16 x^2

A_3 = Area of semicircle to be subtracted = $\frac{\pi R^2}{2} = \frac{\pi(4x)^2}{2} = 25.13x^2$

A_4 = Area of quatercircle to be subtracted = $\frac{\pi R^2}{4} = \frac{\pi(4x)^2}{4} = 12.56x^2$

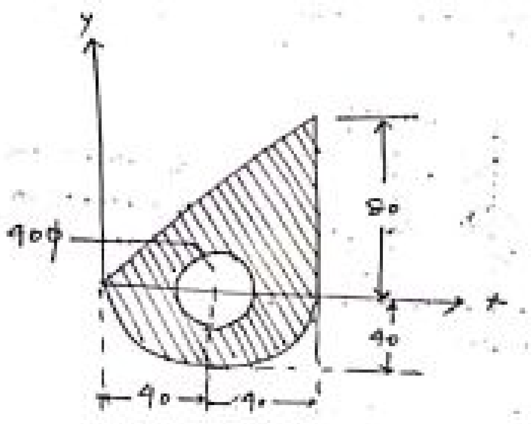
A_5 = Area of triangle = $\frac{1}{2} \times 6x \times 4x = 12x^2$

Area (A_i)	x_i	y_i	$A_i x_i$	$A_i y_i$
$A_1 = 268800$	$7x = 280$	$6x = 240$	75264000	64512000
$A_2 = 25600$	$2x = 80$	$10x = 400$	2048000	10240000
$A_3 = 40208$	$6x = 240$	$\frac{4 \times 4x}{3\pi} = 67.906$	9649920	2730364.448
$A_4 = 20096$	$10x + \left(4x - \frac{4 \times 4x}{3\pi}\right) = 492.09$	$8x + \left(4x - \frac{4 \times 4x}{3\pi}\right) = 412.093$	9889040.64	8281420.926
$A_5 = 19200$	$14x + \frac{6x}{3} = 16x = 640$	$\frac{4x}{3} = 53.33$	12288000	1023936

$$x_c = \frac{A_1x_1 - A_2x_2 - A_3x_3 - A_4x_4 + A_5x_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 326.404mm$$

$$y_c = \frac{A_1y_1 - A_2y_2 - A_3y_3 - A_4y_4 + A_5y_5}{A_1 - A_2 - A_3 - A_4 + A_5} = 219.124mm$$

Problem 6: Determine the centroid of the following figure.



A_1 = Area of triangle = $\frac{1}{2} \times 80 \times 80 = 3200m^2$

A_2 = Area of semicircle = $\frac{\pi d^2}{8} - \frac{\pi R^2}{2} = 2513.274m^2$

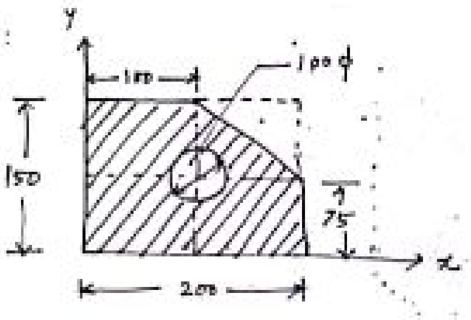
A_3 = Area of semicircle = $\frac{\pi D^2}{2} = 1256.64m^2$

Area (A_i)	x_i	y_i	$A_i\, x_i$	$A_i\, y_i$
3200	2×(80/3)=53.33	80/3 = 26.67	170656	85344
2513.274	40	$\frac{-4\times 40}{3\pi} = -16.97$	100530.96	-42650.259
1256.64	40	0	50265.6	0

$$x_c = \frac{A_1x_1 + A_2x_2 - A_3x_3}{A_1 + A_2 + A_3} = 49.57mm$$

$$y_c = \frac{A_1y_1 + A_2y_2 - A_3y_3}{A_1 + A_2 - A_3} = 9.58mm$$

Problem 7: Determine the centroid of the following figure.



A_1 = Area of the rectangle
A_2 = Area of triangle
A_3 = Area of circle

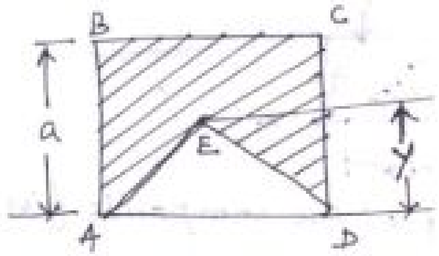
Area (A_i)	x_i	y_i	$A_i\, x_i$	$A_i\, y_i$
30,000	100	75	3000000	2250000
3750	100+200/3 = 166.67	75+150/3 =125	625012.5	468750
7853.98	100	75	785398	589048.5

$$x_c = \frac{\sum A_i x_i}{\sum A_i} = \frac{A_1x_1 - A_2x_2 - A_3x_3}{A_1 - A_2 - A_3} = 86.4mm$$

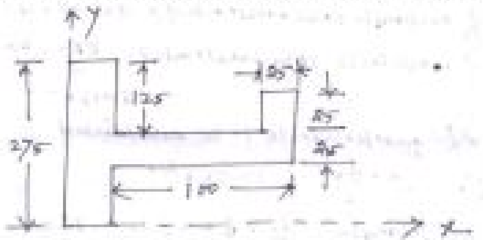
$$y_c = \frac{\sum A_i y_i}{\sum A_i} = \frac{A_1y_1 - A_2y_2 - A_3y_3}{A_1 - A_2 - A_3} = 64.8mm$$

Numerical Problems (Assignment)

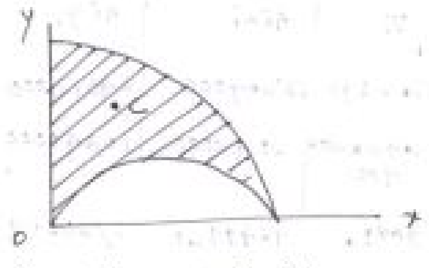
1. An isosceles triangle ADE is to cut from a square ABCD of dimension 'a'. Find the altitude 'y' of the triangle so that vertex E will be centroid of remaining shaded area.



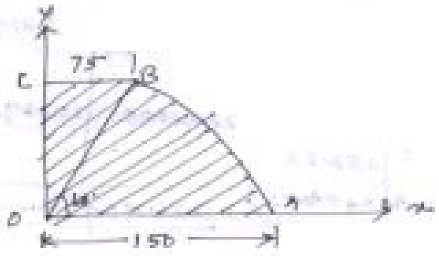
2. Find the centroid of the following figure.



3. Locate the centroid C of the shaded area obtained by cutting a semi-circle of diameter 'a' from the quadrant of a circle of radius 'a'.

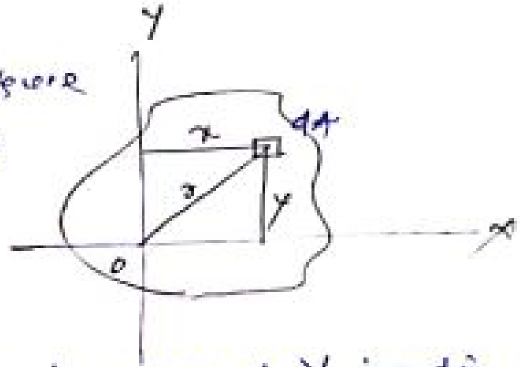


4. Locate the centroid of the composite figure.



Module –I

Moment of Inertia of Plane figures



The moment of inertia of any plane figure with respect to x and y axes in its plane are expressed as

$$I_{xx} = \int y^2 \, dA \qquad I_{yy} = \int x^2 \, dA$$

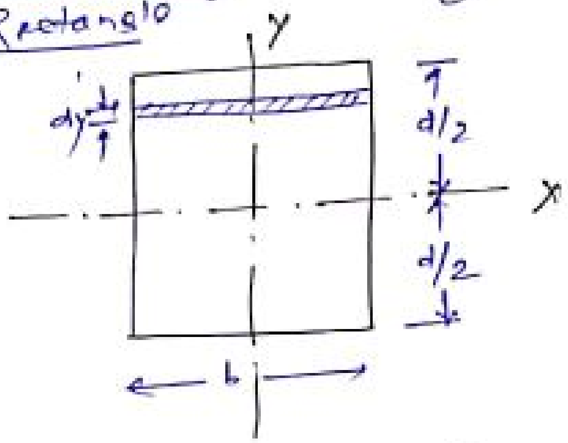
- Ixx and Iyy are also known as second moment of ~~inertia~~ area about the axes as it is distance is squared from corresponding axis.

Unit

Unit of moment of inertia of area is expressed as m^4 or mm^4.

Moment of inertia of Plane figures:-

(i) **Rectangle**



Considering a rectangle of width b and depth d.
Moment of inertia about centroidal axis x-x parallel to the short side i.e b
Now considering an elementary strip of width dy

Moment of inertia of the elemental strip about centroidal axis xx is

$$I_{xx} = y^2 \, dA$$
$$= y^2 \, b\,dy$$

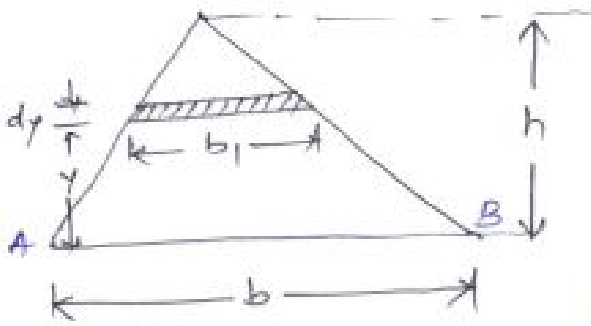
So moment of inertia of entire ~~figure~~ area

$$I_{xx} = \int_{-d/2}^{d/2} y^2 \, b\,dy = b \left| \frac{y^3}{3} \right|_{-\frac{d}{2}}^{d/2} = b\left[\frac{d^3}{24} + \frac{d^3}{24}\right]$$

$$\Rightarrow \boxed{I_{xx} = \frac{bd^3}{12}}$$

Similarly ~~moment as~~ $\boxed{I_{yy} = \frac{db^3}{12}}$

(ii) Triangle :- (Moment of inertia of a triangle about it's b



Consider a small elementary str. at a distance y from the base of thickness dy. Let dA is the area of strip

$dA = b_1 dy$

And $b_1 = \frac{(h-y)}{h} \times b$.

Moment of inertia of strip about base AB

$= y^2 dA = y^2 b_1 dy$

$= y^2 \frac{(h-y)}{h} b\, dy$

∴ Moment of inertia of the triangle about AB

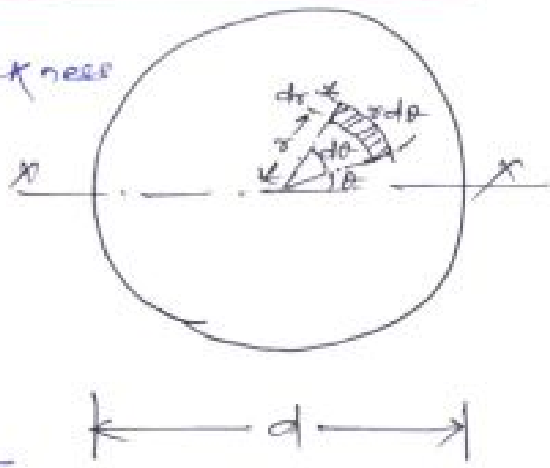
$$I_{AB} = \int_0^h \frac{y^2(h-y)b\,dy}{h} = \int_0^h \left(y^2 - \frac{y^3}{h}\right) b\,dy$$

$$= b\left[\frac{y^3}{3} - \frac{y^4}{4h}\right]_0^h = b\left[\frac{h^3}{3} - \frac{h^4}{4h}\right]$$

$$= b\left[\frac{h^3}{3} - \frac{h^3}{4}\right] = \frac{bh^3}{12}$$

$$\Rightarrow \boxed{I_{AB} = \frac{bh^3}{12}}$$

(iii) Moment of inertia of a circle about it's centroidal axis



Considering an elementary strip of thickness dr, the side of strip $r\,d\theta$.

moment of inertia of strip about xx

$= y^2 dA$

$= (r\sin\theta)^2 r\,d\theta\,dr$

$= r^3 \sin^2\theta\, d\theta\, dr$

∴ Moment of inertia of circle about xx axis

$$I_{xx} = \int^R \int^{2\pi} r^3 \sin^2\theta\, d\theta\, dr$$

$$= \int_0^R \int_0^{2\pi} r^3 \left(\frac{1-\cos 2\theta}{2}\right) d\theta\, dr$$

$$= \int_0^R \frac{r^3}{2}\left[\theta - \frac{\sin 2\theta}{2}\right]_0^{2\pi} dr$$

$$= \int_0^R \frac{r^3}{2}\left(2\pi - \frac{\sin 4\pi}{2}\right) dr$$

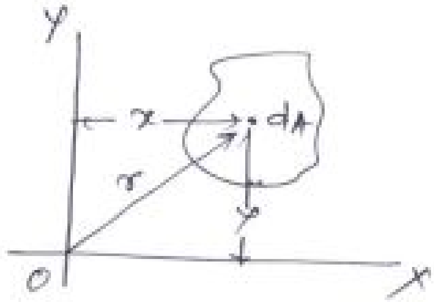
$$= \left[\frac{r^4}{8}\right]_0^R \left[2\pi - 0\right]$$

$$= \frac{R^4}{8} 2\pi = \frac{\pi R^4}{4}$$

$$\Rightarrow \boxed{I_{xx} = \frac{\pi R^4}{4} = \frac{\pi D^4}{64}} \quad \left(\because R = \frac{D}{2}\right)$$

Polar moment of inertia:-

Moment of inertia about an axis perpendicular to the plane of area is called polar moment of inertia. It may denoted as J or I_{zz}



$$\boxed{I_{zz} = \Sigma r^2 dA}$$

Radius of Gyration:-

Radius of gyration may be defined by a relation

$$\boxed{K = \sqrt{\frac{I}{A}}}$$

where K = radius of gyration

I = moment of inertia

A = cross-sectional area

so, we can have the following relations

$$\boxed{\begin{aligned} K_{xx} &= \sqrt{\frac{I_{xx}}{A}} \\ K_{yy} &= \sqrt{\frac{I_{yy}}{A}} \\ K_{AB} &= \sqrt{\frac{I_{AB}}{A}} \end{aligned}}$$

Theorems of Moment of inertia

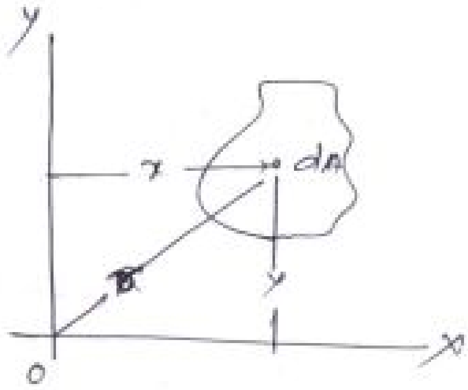
There are two theorems of moment of inertia

(a) perpendicular axis theorem

(b) parallel axis theorem.

Perpendicular axis theorem!-

Moment of inertia of an area about an axis ⊥r to it's plane at any point O is equal to the sum of moments of inertia about any two mutually perpendicular axis through the same point O and lying in the plane of area.



$$I_{zz} = I_{xx} + I_{yy}$$

$$I_{zz} = \Sigma r^2 dA$$

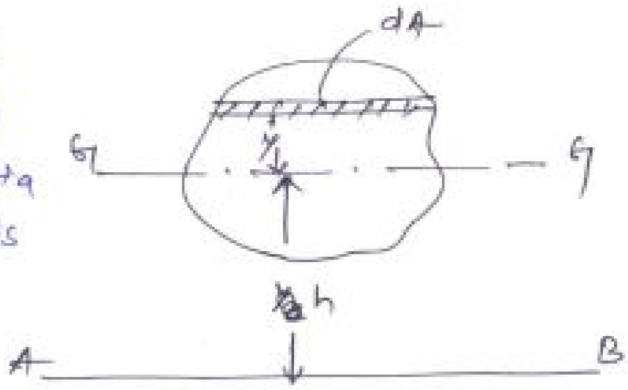
$$= \Sigma (x^2 + y^2)\, dA$$

$$= \Sigma x^2 dA + \Sigma y^2 dA$$

$$\Rightarrow \boxed{I_{zz} = I_{xx} + I_{yy}}$$

Parallel axis theorem!-

Moment of inertia about an axis in the plane of an area is equal to the sum of moment of inertia about a parallel centroidal axis and the product of area and square of the distance betn the two parallel axes.

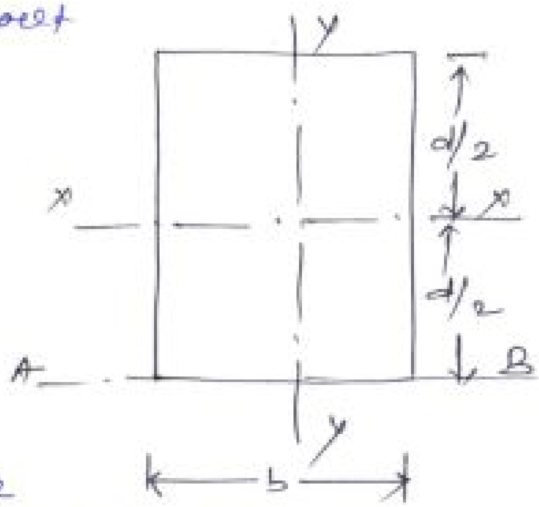


$$\boxed{I_{AB} = I_{GG} + Ah^2}$$

Moment of inertia of standard sections:-

i) Moment of inertia of a rectangle about its centroidal axis xx

$I_{xx} = \frac{bd^3}{12}$



Similarly moment of inertia about it's centroidal axis yy

$I_{yy} = \frac{db^3}{12}$

Now moment of inertia of rectangle about it's base AB can be obtained by applying parallel axis theorem

$I_{AB} = I_{xx} + Ah^2$

$= \frac{bd^3}{12} + (bd)\left(\frac{d}{2}\right)^2$

$= \frac{bd^3}{12} + \frac{bd^2}{4}$

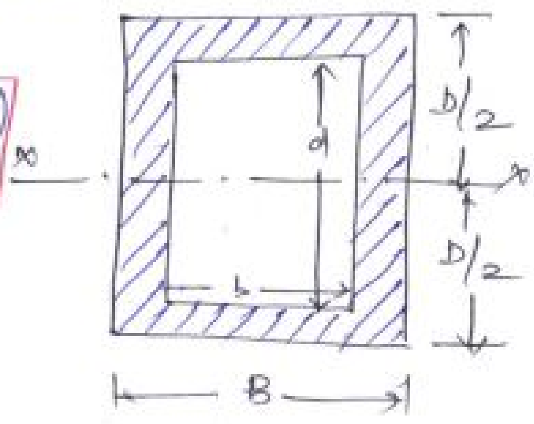
$= \frac{3bd^3 + bd^3}{12} = \frac{bd^3}{3}$

$\Rightarrow \boxed{I_{AB} = \frac{bd^3}{3}}$

(ii) Moment of inertia of a hollow rectangular section:-

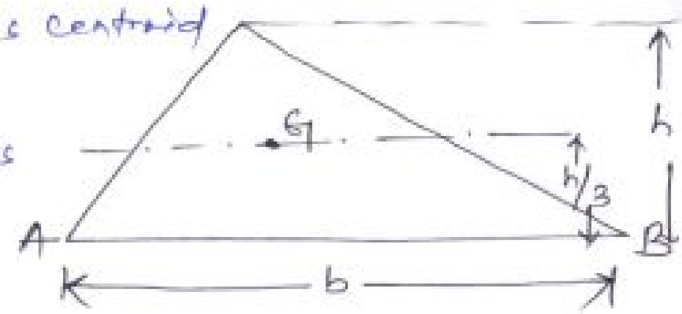
Moment of inertia of hollow rectangular section

$\boxed{I_{xx} = \frac{BD^3}{12} - \frac{bd^3}{12} = \frac{1}{12}\left(BD^3 - bd^3\right)}$



(iii) Moment of inertia of triangle about it's ~~base~~ centroidal

Moment of inertia of triangle about it's base
= moment of inertia about its centroid
+ Ah^2
(using parallel axis theorem)



$\Rightarrow I_{AB} = I_{xx} + Ah^2$

$\Rightarrow \frac{bh^3}{12} = I_{xx} + \frac{1}{2} b \times h \times \left(\frac{h}{3}\right)^2$

$= I_{xx} + \cancel{\frac{bh^3}{2}} \frac{bh^3}{18}$

$\Rightarrow I_{xx} = \frac{bh^3}{12} - \frac{bh^3}{18} = \cancel{\frac{6bh^3 - bh^3}{18}}$

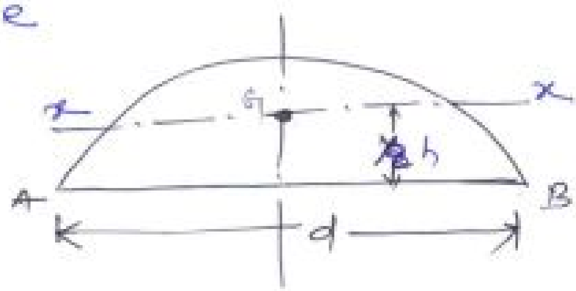
$= \frac{3bh^3 - 2bh^3}{36} = \frac{bh^3}{36}$

$\Rightarrow \boxed{I_{xx} = \frac{bh^3}{36}}$

(iv) Moment of inertia of semicircle
(a) about diametral axis

Moment of inertia of semicircle about $AB = \frac{1}{2} \frac{\pi d^4}{64}$

$= \boxed{\frac{\pi d^4}{128}}$



(b) about centroidal axis xx

$\boxed{h = \frac{4R}{3\pi} = \frac{2d}{3\pi}}$

area $A = \frac{1}{2} \frac{\pi d^2}{4} = \frac{\pi d^2}{8}$

Using parallel axis theorem

$I_{AB} = I_{xx} + Ah^2$

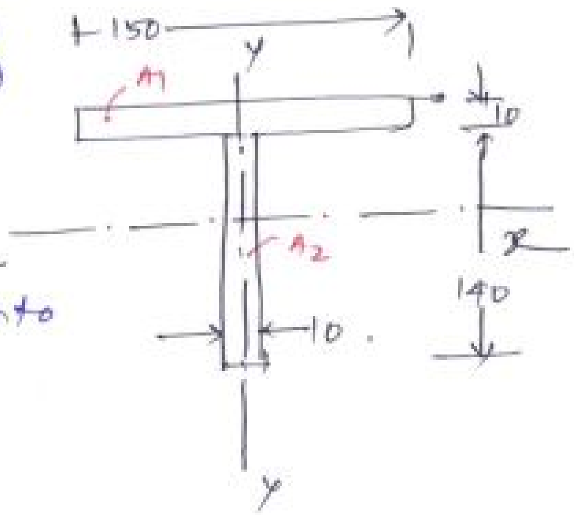
$\Rightarrow \frac{\pi d^4}{128} = I_{xx} + \frac{\pi d^2}{8} \times \left(\frac{2d}{3\pi}\right)^2 = \cancel{\frac{\pi d^3}{128}}$

$$\Rightarrow \frac{\pi d^4}{128} = I_{xx} + \frac{\pi d^2}{8} \times \frac{4d^2}{9\pi^2}$$

$$= I_{xx} + \frac{\pi d^4}{18\pi}$$

$$\Rightarrow \boxed{I_{xx} = \left(\frac{\pi d^4}{128} - \frac{d^4}{18\pi}\right)}$$

Moment of inertia of composite figures :-



Q.1 Determine the moment of inertia of the composite section about an axis passing through the centroidal axis. Also determine MI about axis of symmetry and radius of gyrat'n

Solun Dividing the composite area into A_1 and A_2

$A_1 = 150 \times 10 = 1500\ mm^2$

$A_2 = 140 \times 10 = 1400\ mm^2$

Distance of centroid from base of the composite figure

$$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{(A_1 + A_2)} = \frac{1500 \times 145 + 1400 \times 70}{2900}$$

$$= 108.79\ mm$$

Moment of inertia of the area about xx axis

$$I_{xx} = \left\{\frac{150 \times 10^3}{12} + 1500 \times (145 - 108.79)^2\right\}$$

$$+ \left\{\frac{10 \times 140^3}{12} + 1400 \times (108.79 - 70)^2\right\}$$

$$= (12500 + 1966746.15) + (2286666.667 + 2106529.74)$$

$$= 6372442.557\ mm^4$$

Similarly

$$I_{yy} = \frac{10 \times 150^3}{12} + \frac{140 \times 10^3}{12} = 2812500 + 11666.66667$$

$$= 2824166.667\ mm^4$$

Radius of gyration $k = \sqrt{\frac{I}{A}}$

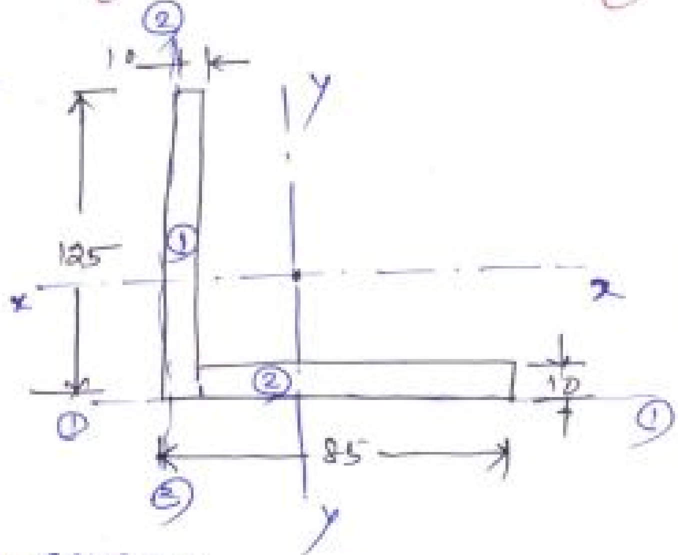
so $k_{xx} = \sqrt{\frac{I_{xx}}{A}}$

$= \sqrt{\frac{6372442.5}{2900}} = 46.87$ mm

Similarly $k_{yy} = \sqrt{\frac{I_{yy}}{A}} = \sqrt{\frac{2824166.667}{2900}}$

$= 31.206$ mm (Ans)

Q.2 Determine the MI of L-section about its centroidal axis parallel to the legs. Also find the polar moment of inertia.



We have $A_1 = 125 \times 10 = 1250$ mm^2

$A_2 = 75 \times 10 = 750$ mm^2

Total area $A_1 + A_2 = 2000$ mm^2

Distance of centroid from 1-1 axis

$\bar{y} = \frac{A_1 y_1 + A_2 y_2}{A_1 + A_2}$

$= \frac{1250 \times 62.5 + 750 \times 5}{2000} = 40.9375$ mm

Distance of centroidal axis yy from 2-2 axis

$\bar{x} = \frac{A_1 x_1 + A_2 x_2}{A_1 + A_2}$

$= \frac{1250 \times 5 + 750 \times \left(\frac{75}{2} + 10\right)}{2000}$

$= \frac{1250 \times 5 + 750 \times 47.5}{2000} = 20.93$ mm

Moment of inertia about xx axis

$I_{xx} = \left\{\frac{10 \times 125^3}{12} + 1250 \times (62.5 - 40.9375)^2\right\}$

$+ \left\{\frac{75 \times 10^3}{12} + 750 \times (40.9375 - 5)^2\right\}$

$= (1627604.167 + 581176.7578) + (6250 + 968627.9297)$

$= \boxed{3183658.854 \text{ mm}^4}$

02/12/2014 ⑤

Similarly MI about yy centroidal axis

$$I_{yy} = \left\{\frac{125\times10^3}{12} + 1250\times(20.93-5)^2\right\} + \left\{\frac{10\times75^3}{12} + 750\times(47.5-20.93)^2\right\}$$

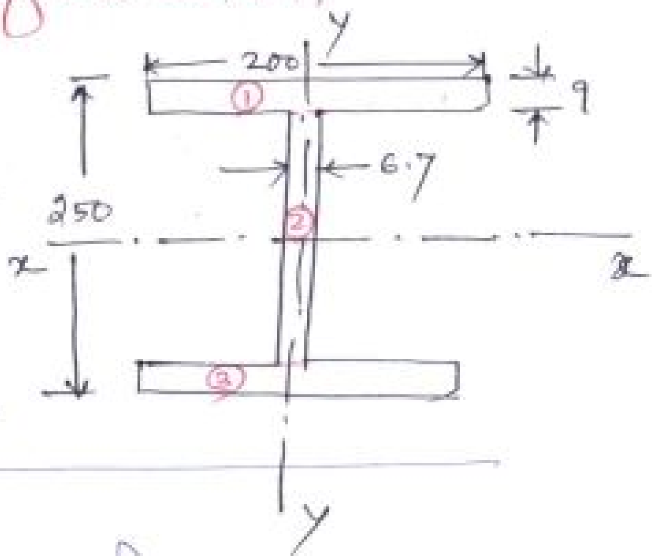
$$= (10416.66667 + 317206.125) + (351562.5 + 529473.675)$$

$$= \boxed{1208658.967\ mm^4}$$

Polar moment of inertia $I_{zz} = I_{xx} + I_{yy}$

$$= \boxed{4392317.821\ mm^4} \quad (\text{Ans})$$

Q.3 Determine the MI of the symmetrical I section about it's centroidal axes x-x and y-y. Also determine the polar moment of inertia of the section.



We have from the figure

$A_1 = 200\times9 = 1800\ mm^2$

$A_2 = 232\times6.7 = 1554.4\ mm^2$

$A_3 = 200\times9 = 1800\ mm^2$

~~Position of centroidal axis xx from base~~

$$\bar{y} = \frac{A_1y_1 + A_2y_2 + A_3y_3}{(A_1+A_2+A_3)}$$

$$= \frac{1800\times(4.5+232+9) + 1554.4\times(\frac{232}{2}+9) + 1800\times4.5}{(1800+1554.4+1800)}$$

$$= \frac{1800\times245.5 + 1554.4\times125 + 1800\times4.5}{(1800+1554.4+1800)}$$

$$= 125\ mm.$$

$$\bar{x} = \frac{A_1x_1 + A_2x_2 + A_3x_3}{(A_1+A_2+A_3)}$$

$$= \frac{1800\times100 + 1554.4\times96.65 + 1800\times100}{(1800+1554.4+1800)} = 98.98$$

MI about xx axis

$$I_{xx} = \left\{ \frac{200 \times 9^3}{12} + 1800 \times (125-4.5)^2 \right\} + \left\{ \frac{6.7 \times 232^3}{12} + 1554.4 \times (\right.$$

$$+ \left\{ \frac{200 \times 9^3}{12} + 1800 \times (125-4.5)^2 \right\}$$

$$= (12150 + 26136450) + (6972002.133 + 0)$$

$$+ (12150 + 26136450)$$

$$= 26148600 + 6972002.133 + 26148600$$

$$= \boxed{59269202.13 \text{ mm}^4}$$

MI about yy axis

$$I_{yy} = \frac{9 \times 200^3}{12} + \frac{232 \times 6.7^3}{12} + \frac{9 \times 200^3}{12}$$

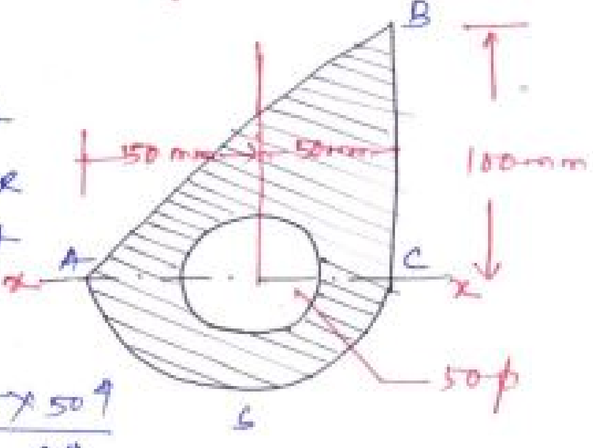
$$= 6000000 + 5814.751 + 6000000$$

$$= \boxed{12005814.75 \text{ mm}^4}$$

Polar moment of inertia $I_{xz} = I_{xx} + I_{yy}$

$$= \boxed{71275016.88 \text{ mm}^4}$$

Q.4 Calculate the moment of inertia of the shaded area about xx axis.



MI of the shaded section about xx = MI of triangle ABC about xx + MI of semicircle ACG about xx − MI of circle

$$= \frac{100 \times 100^3}{12} + \frac{\pi \times 100^4}{128} - \frac{\pi \times 50^4}{64}$$

$$= 8333333.333 + 2454369.261 - 306796.1576$$

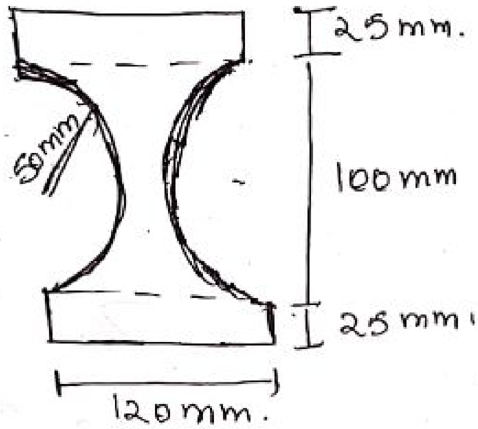
$$= 10480906.44 \text{ mm}^4$$

$$= \boxed{1.048 \times 10^7 \text{ mm}^4}$$

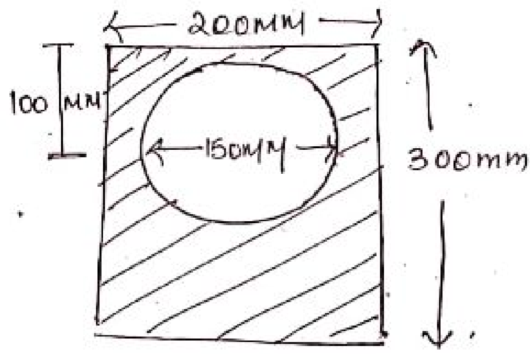
Scanned by CamScanner

Questions

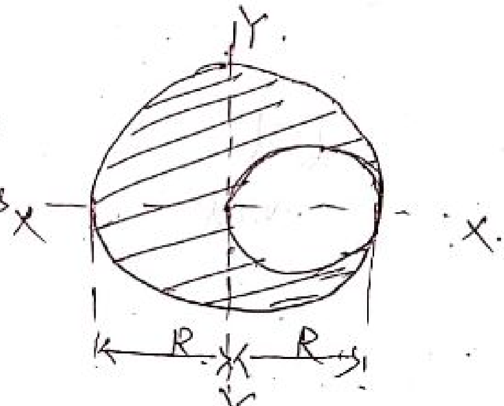
Q. Determine the moments of inertia of the section about horizontal and vertical axes passing through the centroid of the section.



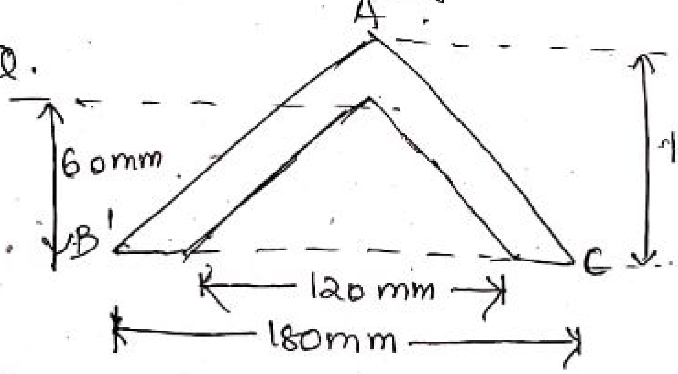
Q. Find the moment of inertia of a hollow section shown in the figure about an axis passing through its centre of gravity or parallel X-axis.



Q. Find the moment of inertia with a circular hole of 30mm diameter about the axis AB as shown in the figure.



Q.



A hollow triangular section is shown in the figure symmetrical about its vertical axis. Find the Moment of inertia of the section about the base "BC".